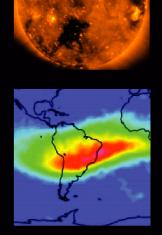


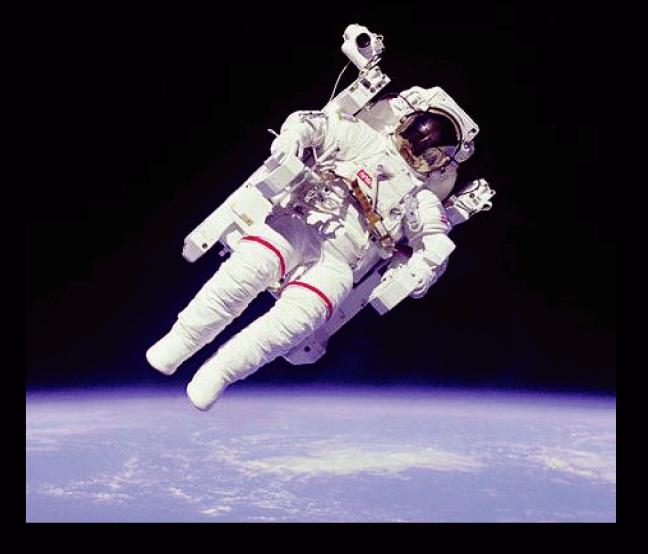
# Math











This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during 2004-2008 school years. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 12. The problems are designed to be 'one-pagers' consisting of a Student Page, and Teacher's Answer Key. This compact form was deemed very popular by participating teachers.

The topic for this collection is **Radiation** which tends to be a very mysterious subject among students and adults. It shouldn't be. Living on the surface of Earth, we are bathed in a natural background of radiation from many sources. Usually the term is identified by the harmful sources (nuclear bombs and power plants), and health risks that occur when too much radiation is absorbed, such as from skin cancer and UV over-exposure, or medical X-rays, CAT scans and cancer therapy. This book will make the student familiar with the many forms of radiation, how it is measured, and what different doses can lead to over time.

This booklet was created by the NASA Space Math program

Dr. Sten Odenwald (NASA - Hinode)
Dr. Neil Zapp (Space Radiation and Analysis Group)

For more weekly classroom activities about astronomy and space science, visit

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at sten.f.odenwald@nasa.gov

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<u>Cover and back page art:</u> Hand X-ray - Dr. David Nelson; Astronaut McCandless EVA (NASA); Galaxy Montage (Chandra); W49-B SNR (Chandra); All Sky gamma-ray map (CGRO-EGRET); X-ray Sun (Hinode).

Individual page illustrations: 1) Pie chart (Lawrence Berkeley Lab wallchart); 2, 3)Pie chart (Beth Israel Deacones Medical Center); 4) Astronaut White spacewalk (NASA / Gemini); 5)Radon gas in USA (EPA); 8) Astronaut radiation dosages (NASA); 9) Van Allen belt (NASA/SRAG) Radiation dosages (NASA/CRRES); 10) Solar images (NASA/ESA/SOHO); 11) Mars radiation (NASA/JPL-Mars Observer); 13) Altair UAV (NASA/Dryden); 17, 18, 19) Earth (NASA/Apollo 17);

<u>Teacher Notes.</u> Here is the order in terms of the mathematics topics covered, and within each topic area, the level of difficulty:

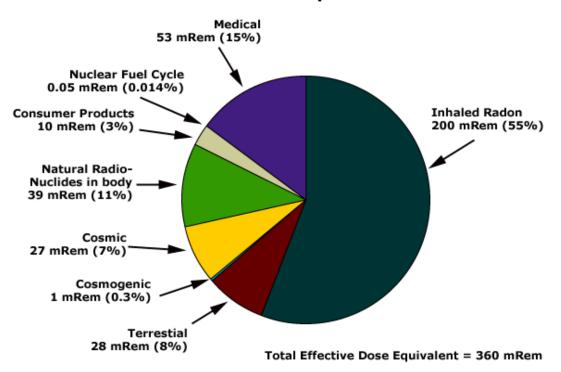
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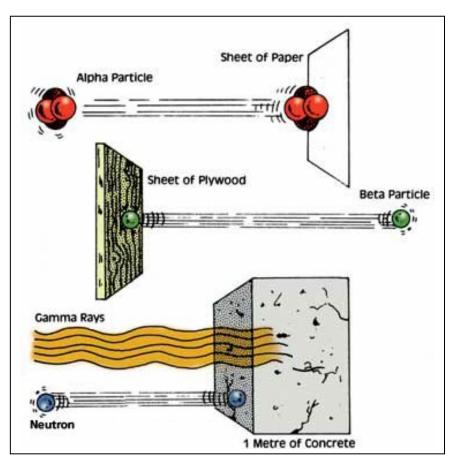
# **Alignment with Mathematics Standards**

The following table connects the activities in this booklet to topics commonly covered in pre-algebra, algebra and calculus textbooks. The cells are shaded according to these three math content areas.

Topic							F	Prok	olen	1	Nur	nbe	r						
_	1	2	3	4	5	6	7	8	9	1	1	1	1	1	1	1	1	1	1
Multiplication and division	Х	X	X	X	X				X	0 X	1 X	2 X	3 X	4	5 X	6 X	7 X	8 X	9 X
Number Patterns														X					
Area, and probability										X									
Venn Diagrams percentages										X									
Averaging							X	X		X									
Scale drawings																			
Polygonal Areas				X					X		X	X	X						
Geometry																	X		
Sci. Notation													X						
Unit Conversions		x	X		X			X	X		X				X			X	X
<b>Graph Analysis</b>				X			X	X	X	Х	X	X	X		X	Х	X	X	
Exponential Functions																X			
Piecewise Functions																Х			
Sin, Cos, Tan																			
Solving for X						X											X		
Quadratic Equations						X													
Evaluating Fns																	X	X	X
Polynomials																			
Function Differentiation																			
Graphical Integration				X					X		X	X							
Function Integration																X			X
Compound Interest																			

#### Sources of Exposure





Different types of radiation can be shielded by different materials

# An Introduction to Space Radiation

Believe it or not, you are surrounded by radiation! As you are sitting here reading this article, electromagnetic radiation from sunlight, electric lights, power cables in the walls, and the local radio station are coursing through your body.

Is radiation something to worry about? It all depends on how much you absorb, and in what forms.

There are two main types of radiation: <u>electromagnetic</u> radiation, and <u>particle</u> radiation. Both forms carry energy, which means that if you accumulate too much over time, either in the tissues of your body, or in sensitive electronic equipment, they can potentially do damage.

Electromagnetic Radiation: This 'EM' radiation travels at the speed of light, and is defined by its wavelength or frequency. EM radiation spans a wide wavelength range from gamma-rays and X-rays at the shortest, through ultraviolet and visible light, and to longer wavelengths in the infrared, and radio. The harmfulness of this radiation depends on BOTH its intensity and its wavelength. A small amount of short-wavelength (UV-B) ultraviolet radiation can give you a nice tan. Too much can increase your risk for skin cancer. A small amount of radio radiation is enough for a car radio to pick up a distant station, but too much in a microwave oven will cook you in 60-seconds flat!

Particle Radiation: This radiation consists of particles of matter traveling through space at high-speed. Usually these particles are atomic particles such as electrons, protons, neutrons, alpha particles or even entire atomic nuclei. A small amount of particle radiation produced by, say, the radium dial of a watch, is enough to make it glow in the dark harmlessly, but too much can destroy the DNA in your cells and lead to mutations, cancer, and even death.

EM radiation is produced by heated bodies, chemical reactions, nuclear reactions and a variety of man-made technologies such as medical imaging systems, radio transmitters, electrical power systems, cell phones, microwave ovens and computers.

Particle radiation is usually produced by unstable atoms that are 'radioactive', or by many different astronomical systems in which matter can be accelerated to high speed such as supernova explosions, pulsars and solar flares. Man-made particle radiation can be produced in nuclear reactors, or under laboratory conditions at Fermi Lab, CERN or other accelerator labs.

There are three different kinds of particle radiation, each produces its own level of tissue damage.

Alpha-particles are given-off by radioactive atoms. They are nuclei containing two protons and two neutrons: essentially helium nuclei. These particles, at high energy, can be very destructive to tissue as they leave tracks of ionization inside cells, as they are slowed-down and give up their kinetic energy. They can be stopped or <a href="mailto:shielded">shielded</a>, by a sheet of paper and rendered harmless, however they can act like dust particles and be inhaled, doing damage to lung tissue as they build up their concentration. Spacesuits can typically shield an astronaut from alpha-particles that occur in solar flares and cosmic rays.

Beta-particles (or Beta-rays) are also given off by radioactive atoms during the process of 'beta decay'. They consist of energetic electrons traveling at high-speed, and require several millimeters of aluminum to stop most of them. A spacesuit normally has a few millimeters of aluminum in its fabric, so astronauts are usually well-protected from beta-rays from solar flares and other sources.

Neutron particles are produced in nuclear reactions including fission and fusion. Because they carry no charge, they easily penetrate many substances. Large quantities of dense lead or cement are required to shield against neutron radiation. Only the walls of spacecraft provide adequate shielding from neutron particles. During EVAs, astronauts receive unavoidable radiation exposure to neutrons.

To discuss radiation and human exposure to it, we have to use standard units to describe the amount present, and its accumulation in both human tissue, and in technological systems.

Scientists measure radiation dosages and exposure in terms of units called Rads and Rems (Grays and Seiverts are used in Europe).

<u>Exposure</u>: This is a measure of the amount of energy that is absorbed by matter over a period of time. This matter can be human tissue, or sensitive computer circuitry. The unit for exposure is the Rad, which means 'Radiation Equivalent Dose'. One Rad is equal to 100 ergs of energy delivered to one gram of matter. The equivalent SI unit is the Gray (G). **One Gray 100 Rads**.

<u>Dosage</u>: This compares the amount of absorbed energy (Rads) to the amount of tissue damage it produces in a human. It is measured in units of the Rem, which means 'Roentgen Equivalent Man'. The equivalent SI unit is the Seivert (Sv). One Seivert = 100 Rems.

Radiation exposure can be very accurately measured and defined. It is just the amount of energy (in ergs or joules) delivered to a sample of matter. Dosage, however, is much more complicated. This term has to do with the amount of damage that a given amount of energy does to a tissue sample or an electronic component. Each kind of radiation, for the same exposure level, produces a different amount of damage. Mathematically, this is represented by the equation:

#### Exposure = Dosage x Q

Because of the Q-factor, different forms of radiation produce different levels of tissue damage. EM radiation, such as x-rays and gamma-rays, produce 'one unit' of tissue damage, so for this kind of radiation Q=1, and so 1 Rad = 1 Rem. This is also the case for beta radiation, which has the same Q value. For alpha particles, Q=15-20, and for neutrons, Q=10. That means that an exposure of 10 Rads of radiation (which equals 1000 ergs delivered to 1 gram of matter) produces a dosage of 1 Rem for Q=10.

Beyond this exposure and dosage, radiation also has different effects depending on how much you absorb over different amounts of time. Let's consider two extreme examples where your entire body is 'irradiated': A small dose over a long time, and a big does over a short time.

Strong and Intense! In cancer therapy, small parts of your body are irradiated to kill cancerous cells. This works because radiation transports energy into cellular tissue where it is absorbed, and cancerous cells are very sensitive to heat compared to normal cells. Although patients report nausea and loss of hair, the benefits to destroying cancerous cells far outweighs the collateral effects, which are usually temporary. Typical dosages are about 200 Rems over a few square centimeters, or even 5,000 Rem over a single tumor area! For whole-body dosages, the effects are far worse as the table below shows!

50 - 100 Rems 100 - 200 Rems 200 - 300 Rems 300 - 400 Rems 400 - 500 Rems 500 - 600 Rems 600 -1,000 Rems 5,000 Rems 8,000 Rems 10,000 Rems	No significant illness Nausia, vomiting. 10% fatal in 30 days. Vomiting. 35% fatal in 30 days. Vomiting, diarrhea. 50% fatal in 30 days. Hair loss, fever, hemorrhaging in 3wks. Internal bleeding. 60% die in 30 days. Intestinal damage. 100% lethal in 14 days. Delerium, Coma: 100% fatal in 7 days. Coma in seconds. Death in an hour. Instant death.
---	--

Weak and Long! On the ground, you receive about 0.4 Rem (e.g. 400 milliRem) of natural backround radiation, radiation from all forms of medical testing, what you eat, and where you live. Over the course of your lifetime, say 80 years, this adds up to  $80 \times 0.4 = 32$  Rem of radiation. By far, the biggest contribution comes from radioactive radon gas in your home, which can amount to as much as 0.1 Rem, which yields a lifetime dose of 8 Rem. Some portion of this radiation exposure invariably contributes to the average cancer risk that each and every one of us experiences.

Medical Diagnostic Radiation:							
0.002 Rems	Dental x-ray						
0.010 Rems	Diagnostic chest X-ray						
0.065 Rems	Pelvis/Hip x-ray						
0.150 Rems	Barium enema for colonoscopy						
0.300 Rems	Mammogram						
0.440 Rems	Bone scan						
2 to 10 Rems	CT scan of whole body						

So, the medical impact of radiation depends on the intensity of the dose, whether your whole body or just a few cells are involved, and how long your exposure will be at a given dosage. This makes the calculation of radiation effects a complicated process. 280 Rems all at once is fatal for 35% of people after 30 days, but the same exposure over a 70-year lifespan is only 4 Rems each year, which is easily survivable and harmless.

Just remember that some exposure to radiation is simply part of the price we pay for living on the surface of Earth and eating a well-balanced diet. In fact, because radiation leads to mutations, and mutations lead to evolution, it is entirely reasonable to say that, without radiation, the organic life on this planet would have taken a very different, and much slower, path!

Would you like to check your annual exposure? Visit the American Nuclear Society webpage and take their test at

http://www.ans.org/pi/resources/dosechart/ or use the one at the US Environmental Protection Agency

http://www.epa.gov/radiation/students/calculate.html or the one at the Livermore National Radiation Laboratory

http://newnet.lanl.gov/main.htm

# Questions to ponder.

1- During an accident, a 70 kg person absorbed 1,000 Rem of x-ray radiation.
A) How much energy, in ergs, did the person gain?
B) If 41,600,000 ergs is needed to raise the temperature of 1 gram of water by 1 degree C, how many degrees did the radiation raise the person's body temperature if the human body is mostly water?
2 - Your probability of contracting cancer from the natural background radiation (0.3 Rem/year) depends on your lifetime exposure. From detailed statistics, a sudden 1 Rem increase in dosage causes an 0.08% increase in deaths during your lifetime, but the same dosage spread over a lifetime causes about 1/2 this cancer increase. By comparison, cancer studies show that a typical person has an 20% lifetime mortality rate from all sources of cancer. (see "Radiation and Risk", Ohio State University, http://www.physics.isu.edu/radinf/risk.htm).
A) Consider 10,000 people exposed to radiation. How many natural cancer deaths would you expect to find in such a sample?
B) How much does the natural background radiation contribute to this cancer death rate?
C) Whenever you take a survey of people, there is a built-in statistical uncertainty in how precisely you can make the measurement, which is found by comparing the sample size to the square-root of the number of samples. In polls, this is referred to as the 'margin of error'. For your answer to Problem 2a, what is the range of people that may die from cancer in this population?
D) Compared to your answer to Problem 2B, do you think you would be able to measure the lifetime deaths from natural background radiation exposure compared to the variation in cancer mortality in this population?

- 1- During an accident, a 70 kg person absorbed 1,000 Rem of x-ray radiation.
  - A) How much energy, in ergs, did the person gain?

    Answer: For X rays, which are electromagnetic radiation.

Answer: For X-rays, which are electromagnetic radiation, Q = 1 so 1 Rem = 1 Rad. Then, 1000 Rem x 100 ergs/gram x 170 kg x 1000 gm/kg = 170,000,000,000 ergs.

B) If 41,600,000 ergs is needed to raise the temperature of 1 gram of water by 1 degree C, how many degrees did the radiation raise the person's body if the human body is mostly water?

Answer: 1000 Rem x 100 ergs/Rem = 100,000 ergs So, 100,000 ergs/ (41,600,000 ergs/degree C) = 0.002 degrees C.

- 2 Your probability of contracting cancer from the natural background radiation (0.3 Rem/year) depends on your lifetime exposure. From detailed statistics, a sudden 1 Rem increase in dosage causes an 0.08% increase in deaths during your lifetime, but the same dosage spread over a lifetime causes about 1/2 this cancer increase. By comparison, cancer studies show that a typical person has an 20% lifetime mortality rate from all sources of cancer. (see "Radiation and Risk", Ohio State University, http://www.physics.isu.edu/radinf/risk.htm).
- A) Consider 10,000 people exposed to radiation. How many natural cancer deaths would you expect to find in such a sample?

Answer:  $10,000 \times 0.2 = 2,000$  deaths over a lifetime.

- B) How much does the natural background radiation contribute to this cancer death rate?

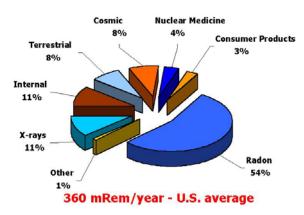
  Answer: 0.3 Rem/yr x 75 years x 0.04% = 0.9% x 10,000 people = 90 people.
- C) Whenever you take a survey of people, there is a built-in statistical uncertainty in how precisely you can make the measurement, which is found by comparing the sample size to the square-root of the number of samples. In polls, this is referred to as the 'margin of error'. For your answer to Problem 2a, what is the range of people that may die from cancer in this population?

Answer:  $(10000)^{1/2}$  = 100 so the range is from (2000 - 100) to (2000 + 100) or 1900 to 2100 people.

D) Compared to your answer to Problem 2B, do you think you would be able to measure the lifetime deaths from natural background radiation exposure compared to the variation in cancer mortality in this population?

Answer: Comparing the 90 deaths to the statistical uncertainty of 100 deaths in a sample of 10,000 people, you would not be able to detect the 90 deaths assigned to the natural background, against the variation of deaths you statistically expect from all other causes of cancer.

## Unit Conversion Exercises



To understand the effect that radiation has on biological systems, a number of different systems for measurement have arisen over the last 50 years. European scientists prefer to use Grays and Seiverts while American scientists still use Rads and Rems!

The chart to the left shows your typical radiation dosage on the ground and the factors that contribute to it.

Basic Unit Conversions:							
1 Curie = 37 billion disintegrations/sec							
1 Gray = 100 Rads	0.001 milli						
1 Rad = 0.01 Joules/kg	0.000001 micro						
1 Seivert = 100 Rems	1 lifetime = 70 years						
1 Roentgen = 0.000258 Charges/kg	1 year = 8760 hours						
1 microCoulomb/kg = 46 milliRem	1 Coulomb = 6.24 billion billion charges						

#### Convert:

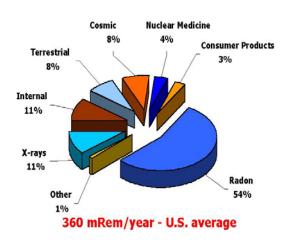
1.	360 milliRem per year tomicroSeiverts per hour
2.	7.8 milliRem per day toRem per year
3.	1 Rad per day toGrays per year
4.	360 milliRem per year toRems per lifetime
5.	3.0 Roentgens to charges per gram
6.	5.6 Seiverts per year tomilliRem per day
7.	537.0 milliGrays per year tomilliRads per hour

360 milliRem/yr x 1Rem/1000 milliRem x 1 year/8760 hours = 0.000041 Rem/hour 0.000041 Rem/hour x 1.0 Seiverts/100 Rem = 0.00000041 Seiverts/hour 0.00000041 Seiverts/hour x 1 microSeivert/0.000001Seivert = 0.41 microSeiverts/hour 2. 7.8 milliRem/day x 365 days/year = 2847.0 milliRem/year 2847.0 milliRem/year x 1.0 Rem/1000milliRem = 2.8 Rem/year 3. 1 Rad/day x 365 days/year x 1 Gray/100 Rads = 3.65 Grays/year 4. 360 milliRem/year x 70 years/lifetime x 1 Rem/1000 milliRem = 25.2 Rems/lifetime 3.0 Roentgens x 0.000258 charges/kg per Roentgen = 0.000774 charges/kg 0.000774 charges/kilogram x 1.0 kg/1000 gram = 0.000000774 charges/gram 6. 5.6 Seiverts/year x 1.0 Year/365 days x 100 Rem/1.0 Seivert = 1.53 Rem/day 1.53 Rem/day x 1000 milliRem/Rem = 1530 milliRem/day 537.0 milliGrays/year x 1.0 years/8760 hours x 100 Rads/1.0 Gray = 6.13 milliRads/hour

Note: There are many different conversion 'chains' that the students can offer. The challenge is to set up each ratio correctly with the right number in the numerator and

denominator!

# Background Radiation and Lifestyles



As we go about our daily lives, we are constantly surrounded by naturallyoccurring sources of radiation. accumulation of this radiation dosage every day throughout our lives leads to our total lifetime dosage. Depending on where we live, and our lifestyles, this lifetime dosage can make us susceptible to various forms of cancers. Generally, the lower your lifetime dose, the lower your risk for cancer.

In the following activity, you will calculate the total lifetime dosages (in Rems) for a person living in several different geographic locations with a variety of lifestyles.

- 1. Nancy was born in Denver where the cosmic rays (GCR) produce 120 milliRem/year and an additional 105 milliRem/year comes from the ground (Terr.). After 30 years, she moves to Baton Rouge, Louisiana where GCR = 35 milliRem/year and Terr. = 40 milliRem/year. At both locations, she buys the same kind of house and she receives 100 milliRem/year from radon gas in the basement. Assuming all other lifestyle sources contribute 50 milliRem/year during her entire life, and that she is now 65 years old, what has been her total radiation dosage to date in Rem?
- 2. Suppose that Nancy was also a cigarette smoker since she was 16 years old, but that she gave up smoking when she turned 52. How much additional lifetime radiation dosage in Rems did she receive from this habit during the time she lived in Denver and Baton Rouge if her one-pack-a-day habit exposed her to 15 milliRem/year?
- 3. Suppose that Nancy was also an airline pilot since she was 27 years old. She has been smoking since age 16. She flys 900 hours each year, with 90% of this time spent at cruising altitudes (35,000 feet) where the cosmic radiation dosage is 5 microSeiverts per hour. If 1 Seivert = 100 Rems, how much additional radiation has she received than in your answer to Question 2?
- 4. Suppose that after 30 years, instead of moving to Baton Rouge, Nancy moved from Denver to Kerala, India where the terrestrial radiation dosage (Terr.) is 380 milliRem/year, but gives up smoking. What will be her total dosage by age 65?
- 5. Instead of being an airline pilot, at age 35 she decides to become a non-smoking astronaut. From Denver, she moved to Baton Rouge for 5 years, and then finds a home in Houston near the NASA Johnson Spaceflight Center, which is the hub of manned spaceflight activities. At this location, GCR = 45 milliRem/year and Terr. = 30 milliRem/year. At age 39 she becomes the co-pilot for the Space Shuttle Atlantis on a 13-day trip, during which time her radiation dosage is 19 milliRem/day. If she takes three of these trips before age 65, what is her total dosage?

1. Nancy was born in Denver where the cosmic rays (GCR) produce 120 milliRem/year and an additional 105 milliRem/year comes from the ground (Terr.). After 30 years, she moves to Baton Rouge, Louisiana where GCR = 35 milliRem/year and Terr. = 40 milliRem/year. At both locations, she buys the same kind of house and she receives 100 milliRem/year from radon gas in the basement. Assuming all other lifestyle sources contribute 50 milliRem/year during her entire life, and that she is now 65 years old, what has been her total radiation dosage to date in Rem?

Denver: (120 + 105 + 100 + 50)millirem/year x 30 years x 1 Rem/1000 milliRems = 11.25 Rem Baton Rouge: (35 + 40 + 100 + 50) millirem/year x (65-30) years x 1 Rem/1000 milliRems = 7.88 Rem

Total = 11.25 Rems + 7.88 Rems = 19.1 Rems.

2. Suppose that Nancy was a cigarette smoker since she was 16 years old, but that she gave up smoking when she turned 52. How much additional lifetime radiation dosage in Rems did she receive from this habit during the time she lived in Denver and Baton Rouge if her one-pack-a-day habit exposed her to 15 milliRem/year?

Smoking = 15 milliRem/year  $\times$  (52-16) years  $\times$  1.0 Rem / 1000 milliRems = 0.5 Rem Geographic = 19.1 Rem

Total = 19.1 Rems + 0.5 Rems = 19.6 Rems

3. Suppose that Nancy was also an airline pilot since she was 27 years old, and retired at 45. She has been smoking since age 16. She flys 900 hours each year, with 90% of this time spent at cruising altitudes (35,000 feet) where the cosmic radiation dosage is 5 microSeiverts per hour. If 1 Seivert = 100 Rems, how much additional radiation has she received than in your answer to Question 2?

900 hours/year  $\times$  (45-27)  $\times$  0.90 = 14,580 hours. 5 microSeiverts/hour  $\times$  100 Rems/1 Seivert = 500 microRems/hour 500 microRems/hour  $\times$  14,580 hours  $\times$  1 Rem/1000000microRem = 7.3 Rems Total = 19.6 Rems + 7.3 Rems = 26.9 Rems

4. Suppose that after 30 years, instead of moving to Baron Rouge, Nancy moved from Denver to Kerala, India where the terrestrial radiation dosage (Terr.) is 380 milliRem/year, but gives up smoking. What will be her total dosage by age 65?

Denver = 11.3 Rems

Kerala = 380 milliRems/year  $\times$  (65-30) years  $\times$  1.0 Rem/1000 milliRems = 13.3 Rems Total = 11.3 Rems + 13.3 Rems = 24.6 Rems

5. Instead of being an airline pilot, at age 35 she decides to become a non-smoking astronaut. After 30 years in Denver, she moves to Baton Rouge for 5 years, then finds a home in Houston. At this location, GCR = 40 milliRem/year and Terr. = 30 milliRem/year. At age 39 she becomes the co-pilot for the Space Shuttle Atlantis on a 13-day trip, during which time her radiation dosage is 19 milliRem/day. If she takes three of these trips before age 65, what is her total dosage?

Denver: 11.3 Rems

Baton Rouge: 225 millirem/year  $\times$  (35-30) years  $\times$  1 Rem/1000 milliRems = 1.1 Rem Houston: 220 millirem/year  $\times$  (65-35) years  $\times$  1 Rem/1000 milliRems = 6.6 Rem

Shuttle Flights:  $3 \times 13$  days  $\times 19$  milliRem/day = 0.7 Rems

Total = 11.3 Rems + 1.1 Rems + 6.6 Rems + 0.7 Rems = 19.7 Rems

Space Math

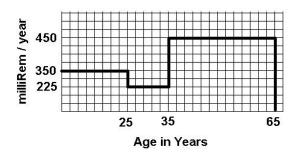
http://spacemath.gsfc.nasa.gov

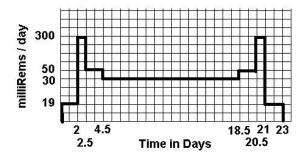


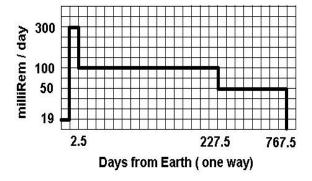
Space travel is understandably a risky business. One of the most well-studied, and worrisome, hazards is the radiation environment. The sun produces streams of high-energy particles and flares, while the universe itself also rains particles down upon us from distant supernova explosions and other energetic phenomena. But how bad is space travel compared to just staying on Earth?

For the following problems, plot how the radiation environment changes for a person living in Denver, on the Space Station, on the Moon, and on a journey to Mars and back. Calculate the total radiation dosage by computing the area under the respective curves.

- 1. Nancy was born and raised in Denver where her radiation dosage was 350 milliRems/year. At age 25, she moved to Houston where her dosage was 225 milliRems/year, then moved to South Dakota 10 years later where her dosage was 450 milliRems/year until she retired at age 65. Create a plot showing 'YEAR' on the horizontal axis and 'Dosage' on the vertical axis. Prove that the product of the vertical axis units times the horizontal axis units is the total dose in milliRems. Plot Nancy's annual dosages and calculate her total dosage by age 65.
- 2. An astronaut travels to the Moon on NASA's Orion Crew Vehicle, and spends two weeks on the lunar surface before returning to Earth. The radiation dosage is 19 milliRem/day in Earth orbit for each of two days. The 1/2 day trip through the van Allen belts is 300 milliRem/day. The journey to the Moon takes two days at 50 milliRem/day. The stay on the lunar surface under shielded conditions is 30 milliRem/day. The astronaut returns to Earth retracing the previous conditions, followed by a 2-day stay at the International Space Station, where the dosage is 1.5 milliRem/hour. Plot her dosage history and calculate the total dosage in Rems.
- **3**. An astronaut journeys to Mars. The radiation dosage is 19 milliRem/day at the International Space Station for each of two days. The 1/2 day trip through the van Allen belts was 300 milliRem/day. The crew spends 225 days traveling to Mars, during which time the dosages are 100 milliRems/day. On Mars, for a planned stay of 540 days, the dosage will be about 50 milliRem/day. This is followed by a similar 225-day return to earth, 1/2-day trip through the van Allen Belts, and a 2-day stay at the Space Station. Plot her dosage history and calculate the dosages.







#### Problem 1:

#### **Denver to Houston to South Dakota:**

(350 mRem/yr x 25 yrs) + (225 mRem/yr x 10 yrs) + (450 mRem/yr x 30 yrs) = **24.8 Rem** 

#### Problem 2:

#### **Roundtrip:**

( 19 mRem/day x 2 days) + (300 mRem/day x 0.5 days) + (50 mRem/day x 2 days) + (30 mRem/day x 14 days) + (50 mRem/day x 2 days) + (300 mRem/day x 0.5 days) + (19 mRem/day x 2 days) = **1.1 Rem** 

#### Problem 3:

#### **Earth to Mars:**

(19 mRem/day x 2 days) + (300 mRem/day x 0.5 days) + (100 mRem/day x 225 days) + (50 mRem/day x 540 days) = 49.7 Rem

Return Trip = 22.7 Rem

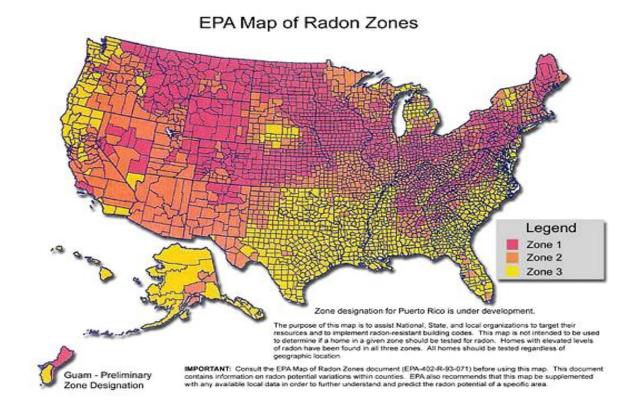
<u>Total Trip = 49.7 Rem + 22.7 Rem = 72.4</u> Rem

#### **Note to Teacher:**

The total lifetime radiation dosages for the trips in Problem 2 and 3 will be in ADDITION to the total dosages that the astronauts receive on the ground before and after the trip into space. For example, if an astronaut lives in Houston all his life (70 years) where the environmental and lifestyle dosage is 300 milliRems/year, the normal lifetime dosage will be 300 milliRems/year x 70 years = 21.0 Rems.

In Problem 3, an astronaut travels to Mars and back, taking (2.5 + 225 + 540 + 225 + 2.5) = 995 days or 2.7 years their total Mars dosage will be 72.4 Rem added to (70-2.7)x 300 milliRems/year = 20.2 Rems on the ground for a total lifetime dosage of 92.6 Rems!

Another way to look at this is to recognize that a trip to Mars will equal about 72.4 Rem/0.300 Rem = 241 years of normal background radiation living on Earth (in Houston)...but accumulated in only 2.7 years!



Most family rooms (dens) are located in the basements of homes across the country. This is also the place where radon gas can collect over time. When inhaled over time, radon gas adds to your lifetime natural background radiation exposure, and is a significant risk factor for various forms of lung and respiratory cancer. This is why in many states, home buyers must have prospective homes tested before purchase. The typical, annual radiation exposure from all non-radon forms of natural exposure is about 200 milliRem per year.

The above figure shows the four radon zones based on a study by the US Environmental Protection Agency (<a href="http://www.epa.gov/radon/zonemap.html">http://www.epa.gov/radon/zonemap.html</a>). By the way, you can also find maps for individual states at this website. The four zones correspond to radiation dosages of Zone 1: 4 picoCuries/liter Zone 2: 3 picoCuries/liter Zone 3: 2 picoCuries/Liter. Note: 4 picoCuries/liter for a full-year exposure is equal to about 3 Rems.

**Problem 1**: A typical family may only spend 4 hours a day in the basement room. What fraction of a full year does this represent?

**Problem 2:** In Zone-1, a full years exposure equals 3 Rem. From your answer to Problem 1, what would you predict as the total annual dosage, in milliRems, for a member of this family if they were living in A) Zone-1? B) Zone-2? C) Zone-3?

**Problem 3:** If a typical lifetime is 80 years, what would be the total lifetime radiation dosage from radon in Rem for the family members in Problem 1 if they lived in A) Zone-1; B) Zone-2; C) Zone-3?

**Problem 1**: A typical family may only spend 4 hours a day in the basement room. What fraction of a full year does this represent?

Answer: (4/24) = 1/6th of a year

**Problem 2:** In Zone-1, a full years exposure equals 3 Rem. From your answer to Problem 1, what would you predict as the total annual dosage, in milliRems, for a member of this family if they were living in:

A) Zone-1?

Answer:  $3 \text{ Rem/year } \times 1/6 \text{ year} = 1/2 \text{ Rem} = 500 \text{ millirem}$ 

B) Zone-2?

Answer:  $3/4 \times 3 \text{ Rem/year} \times 1/6 \text{ year} = 3/8 \text{ Rem} = 375 \text{ milliRem}$ 

C) Zone-3?

Answer:  $2/4 \times 3 \text{ Rem/yr} \times 1/6 \text{ year} = 1/4 \text{ Rem} = 250 \text{ milliRem}$ 

**Problem 3:** If a typical lifetime is 80 years, what would be the total lifetime radiation dosdage from radon in Rem for the family members in Problem 1 if they lived in

- A) Zone-1; Answer =  $80 \times 1/2 \text{ Rem} = 40 \text{ Rem}$
- B) Zone-2; Answer =  $80 \times 3/8 \text{ Rem} = 30 \text{ Rem}$
- C) Zone-3? Answer =  $80 \times 1/4 \text{ Rem} = 20 \text{ Rem}$ .

# Some Puzzling Thoughts about Space Radiation



We have all heard, since grade school, that 1\_\_\_\_\_\_ affects living systems by causing cell mutations. The particles such as fast-moving ions or 2\_\_\_\_\_ strike particular locations in the 3\_\_\_\_\_ of a cell, causing the cell to malfunction, or 4\_\_\_\_ and pass-on a 5\_\_ its progeny. Sometimes the mutations are not beneficial to an organism, or to the evolution of its species. When this happens you can get 6\_\_\_\_ Cancer risks are generally related to the total amount of lifetime radiation exposure. The studies of \_\_\_\_\_ survivors, however, still show that there is much we have to learn about just how radiation delivers its harmful impact. Very large 8\_\_\_\_\_ over a short period of time seem not to have quite the deleterious affect that, say, a small dosage delivered steadily over many years does. The National Academy of Sciences has looked into this issue rather carefully over the years to find a relationship between 9\_\_\_\_\_ cancer risks and low-level radiation exposure. What they concluded was that you get up to 100 cancers per 100,000 people for every 1000 10 \_\_\_\_\_ of additional dosage per year above the natural 11\_\_\_\_\_ rate. If a dosage of 1000 millirems extra radiation per year, adds 100 extra deaths per 100,000, then as little as one extra millirem per annum could cause cancer in one person per 12\_\_\_\_\_\_. Although it's just a 13\_\_\_\_\_\_ estimate, if you happen to be that 'one person' you will be understandably 14\_\_\_\_\_. No scientific study, by the way, has shown that radiation has such a 15\_\_\_\_\_ impact at all levels below 100 millirem, but that's what the \_\_\_\_\_ application of arithmetic shows. Government safety regulations now require that people who work with radiation, such as 17\_\_\_ nuclear medicine technologists, or nuclear power plant operators, are given a maximum permissible dose limit of 500 millirems per year above the prevailing 18\_\_\_\_\_\_ background rate. For you and me doing ordinary work in the office, factory or store, the acceptable maximum dose is 1000 milliRems/year above the 350 milliRem you get each year from natural sources. As a comparison, if you lived within 20 miles of the 19 nuclear power 20\_\_\_\_\_ at the time of its 21\_\_\_\_\_ meltdown, your annual dose would have been about 1500 milliRem/year during the first year, declining slowly as the radioactive 22\_\_\_\_\_ in the environment decay (Excerpted from 'The 23rd Cycle", Sten Odenwald, Columbia University Press)

Solve for X in each equation, and select the correct word from the pair of solutions for X, to fill-in the indicated blanks from 1 to 22 in the essay above.

 $x^2 - 3x - 88 = 0$ 

- $x^2 + 4x 5 = 0$  $x^2 - 3x + 2 = 0$ 3)  $x^2 - x - 12 = 0$ 4)  $2x^2 - 12x + 10 = 0$  $x^2 - 2x - 24 = 0$ 6)  $x^2 + 5x + 6 = 0$  $x^2 - 9 = 0$ 8)
  - x<sup>2</sup> 4x 21 14)  $x^2 - x - 30$ 15)  $x^2 - 9x - 36 = 0$ 16)  $x^2 - 16x + 63 = 0$ 17)  $x^2 + 16x + 63 = 0$ 18)  $x^2 + 14x + 48 = 0$  $x^2 + 19x + 90 = 0$ 19)
  - $2x^2 + 4x 30 = 0$  $x^2 + 8x - 33$ 20)  $3x^2 + 3x - 6$  $x^2 - 100 = 0$ 21)  $x^2 - 8x = 0$  $x^2 - 6x - 16 = 0$ 22)

#### Word bank - factor list

9)

-11 plant -4 cancer 3 lifetime 10 1986 -10 2005 -3 dosages 4 survive -9 Chernobyl -2 Hiroshima 5 mutation 12 linear -8 million -1 radiation 6 upset -7 dentists 0 isotopes 7 statistical -6 natural 1 milliRems 8 background 2 DNA 9 blind -5 neutrons

hundred

#### Here are the correct words added:

We have all heard, since grade school, that **1-radiation** affects living systems by causing cell mutations. The particles such as fast-moving ions or **2-neutrons** strike particular locations in the **3-DNA** of a cell, causing the cell to malfunction, or **4-survive** and pass-on a **5-mutation** to its progeny. Sometimes the mutations are not beneficial to an organism, or to the evolution of its species. When this happens you can get **6-cancer**.

Cancer risks are generally related to the total amount of lifetime radiation exposure. The studies of **7-Hiroshima** survivors, however, still show that there is much we have to learn about just how radiation delivers its harmful impact. Very large **8-dosages** over a short period of time seem not to have quite the deleterious affect that, say, a small dosage delivered steadily over many years does.

The National Academy of Sciences has looked into this issue rather carefully over the years to find a relationship between **9-lifetime** cancer risks and low-level radiation exposure. What they concluded was that you get up to 100 cancers per 100,000 people for every 1000 **10-millirems** of additional dosage per year above the natural **11-background** rate. If a dosage of 1000 millirems extra radiation per year, adds 100 extra deaths per 100,000, then as little as one extra millirem per annum could cause cancer in one person per **12-million**. Although it's just a **13-statistical** estimate, if you happen to be that 'one person' you will be understandably **14-upset**. No scientific study, by the way, has shown that radiation has such a **15-linear** impact at all levels below 100 millirem, but that's what the **16-blind** application of arithmetic shows.

Government safety regulations now require that people who work with radiation, such as **17-dentists**, nuclear medicine technologists, or nuclear power plant operators, are given a maximum permissible dose limit of 500 millirems per year above the prevailing **18-natural** background rate. For you and me doing ordinary work in the office, factory or store, the acceptable maximum dose is 1000 milliRems/year above the 350 milliRem you get each year from natural sources. As a comparison, if you lived within 20 miles of the **19-Chernobyl** nuclear power **20-plant** at the time of its **21-1986** meltdown, your annual dose would have been about 1500 milliRem/year during the first year, declining slowly as the radioactive **22-isotopes** in the environment decay away.

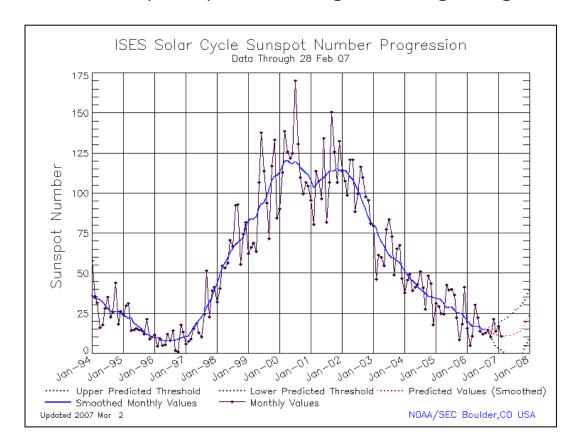
(Excerpted from 'The 23rd Cycle", Sten Odenwald, Columbia University Press)

1)	(x - 3) (x + 1)	3, <b>-1</b>	radiation	12)	(x + 8) (x - 11)	<b>-8</b> , 11	million
2)	(x + 5) (x - 1)	<b>-5</b> , 1	neutrons	13)	(x - 7) (x+3)	<b>7</b> , -3	statistical
3)	(x - 2) (x - 1)	<b>2</b> , 1	DNA	14)	(x - 6) (x + 5)	-5, <b>6</b>	upset
4)	(x - 4) (x + 3)	<b>4</b> ,-3	survive	15)	(x - 12)(x + 3)	<b>12</b> , -3	linear
5)	(2x - 2) (x - 5)	1 , <b>5</b>	mutation	16)	(x - 7) (x - 9)	7, <b>9</b>	blind
6)	(x - 6) (x + 4)	6, <b>-4</b>	cancer	17)	(x + 7) (x + 9)	<b>-7</b> , -9	dentists
7)	(x + 2) (x + 3)	<b>-2</b> , -3	Hiroshima	18)	(x + 6) (x + 8)	<b>-6</b> , -8	natural
8)	(x + 3) (x - 3)	<b>-3</b> , 3	dosages	19)	(x + 10) (x + 9)	-10, <b>-9</b>	Chernobyl
9)	(2x - 6)(x + 5)	<b>3</b> , -5	lifetime	20)	(x + 11) (x - 3)	<b>-11</b> , 3	plant
10)	(3x + 6)(x - 1)	2, <b>1</b>	milliRems	21)	(x + 10) (x - 10)	-10, <b>10</b>	1986
11)	(x - 8) (x + 2)	<b>8</b> , -2	background	22)	x ( x - 8)	<b>0</b> , 8	isotopes

#### Word bank - factor list

4.4	mlant	40	lin a a r
-11	plant	12	linear
-10	2005	11	hundred
-9	Chernobyl	10	1986
-8	million	9	blind
-7	dentists	8	background
-6	natural	7	statistical
-5	neutrons	6	upset
-4	cancer	5	mutation
-3	dosages	4	survive
-2	Hiroshima	3	lifetime
-1	radiation	2	DNA
0	isotopes	1	milliRems

# The Sunspot Cycle - endings and beginnings



The above plot shows the current sunspot cycle (Number 23) based on the average monthly sunspot counts since January, 1994.

Problem 1 - About when (month and year) did Sunspot Cycle 23 begin?

Problem 2 - About when (month and year) did Sunspot Cycle 23 reach its maximum?

<u>Problem 3</u> - A) What was the average minimum sunspot count during the years of the previous sunspot minimum? B) What do you think the average sunspot count will be during the current sunspot minimum?

<u>Problem 4</u> - What is the number of years between sunspot minima to the nearest tenth of a year?

Problem 5 - How long did Cycle 23 take to reach sunspot maximum?

<u>Problem 6</u> - When (year, month) do you predict we will reach sunspot maximum during the next cycle ( Cycle 24)?

Problem 7 - When (year, month) do you think the next sunspot minimum will occur?

<u>Problem 8</u> - During which part of the sunspot cycle is there A) the greatest month-to-month variation in the number of sunspots counted? B) The least variation in the number counted?

<u>Problem 1</u> - When (month and year) did Sunspot Cycle 23 begin? **Answer:** Around July, 1996

Problem 2 - When (month and year) did Sunspot Cycle 23 reach its maximum?

Answer: Around July, 2000 and a second maximum near September, 2001

<u>Problem 3</u> - A) What was the average minimum sunspot count during the previous sunspot minimum?

**Answer:** From the graph the monthly numbers are 5,8,6,6,12,8,13,1,0,16,13,6 for an average of 8 sunspots during 1996.

B) What do you think the average sunspot count will be during the current sunspot minimum?

Answer: About 12.

<u>Problem 4</u> - What is the number of years between sunspot minima to the nearest tenth of a year?

**Answer:** The first minimum was on July, 1996 and the current minimum seems to be around March ,2007 so the difference is 2007.25 - 1996.58 = 10.7 years.

Problem 5 - How long did Cycle 23 take to reach sunspot maximum?

**Answer:** The first maximum occurred on July 2000, the minimum was July 1996, so it took 4 years.

<u>Problem 6</u> - When (year, month) do you predict we will reach sunspot maximum during the next cycle ( Cycle 24)?

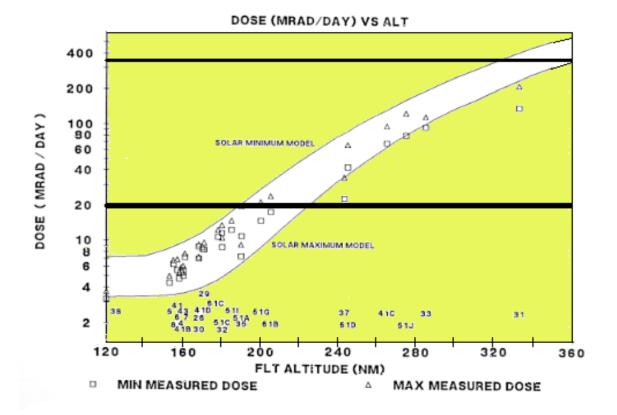
**Answer:** If we add 4 years to the current minimum on March, 2007 we get March, 2011.

Problem 7 - When (year, month) do you think the next sunspot minimum will occur?

**Answer:** From our answer to Problem 4, if we add 10.7 years to March, 2007 we get 2007.25 + 10.7 = 2017.95 or December, 2017.

<u>Problem 8</u> - During which part of the sunspot cycle is there A) the greatest month-to-month variation in the number of sunspots counted? B) The least variation in the number counted?

**Answer:** Looking at the graph, the largest variations from month to month occur near sunspot maximum, and the least variations occur near sunspot minimum.



The typical radiation dosage on the ground is about 1.0 milliRad/day or 360 milliRad/year. This dosage is considered safe, and it is an unavoidable part of the natural background that we live and work within. In space, however, this normal background dosage is significantly exceeded. The figure above shows the radiation dosages encountered by Space Shuttle astronauts during various missions indicated by the numbers near the bottom of the graph. For example, at the far right, astronauts onboard Shuttle Mission STS-31 at an orbital altitude of 335 Nautical Miles (NM), experienced dosages between 150 to 200 milliRads per day.

Problem 1 - At about what altitude do most Space Shuttles orbit?

Problem 2 - What is the average daily dose at this altitude in milliRads/day?

Problem 3 - For a typical Shuttle mission of 10 days, what will be the astronaut's average dose?

Problem 4 - If the astronaut remained on the ground during this mission, how much of a dosage would he have acquired?

Problem 5 - How much radiation dosage did the STS-31 astronauts accumulate during their 118-hour mission to place the Hubble Space Telescope in orbit?

The typical radiation dosage on the ground is about 1.0 milliRad/day or 360 milliRad/year. These dosages are considered safe, and part of the natural background that we live and work within. In space, however, this normal background dosage is significantly exceeded. The figure above shows the radiation dosages encountered by Space Shuttle astronauts during various missions indicated by the numbers near the bottom of the graph. For example, at the far right, astronauts onboard Shuttle Mission STS-31 at an orbital altitude of 335 Nautical Miles (NM), experienced dosages between 150 to 200 milliRads per day.

Problem 1 - At about what altitude do most Space Shuttles orbit?

Answer - The average of the cluster of points is near about 170 Nautical Miles.

Problem 2 - What is the average daily dose at this altitude in milliRads/day?

Answer - At 170 NM, the average dosage is about 9 milliRad/day

Problem 3 - For a typical Shuttle mission of 10 days, what will be the astronaut's average dose?

Answer - 10 days x 9 milliRad/day = 90 milliRads.

Problem 4 - If the astronaut remained on the ground during this mission, how much of a dosage would he have acquired?

Answer - 9 days x 1 milliRad/day = 9 milliRads.

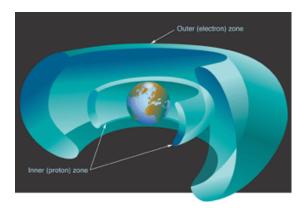
Problem 5 - How much radiation dosage did the STS-31 astronauts accumulate during their 118-hour mission to place the Hubble Space Telescope in orbit? About how many years of ground dosage does this equal?

Answer - The radiation dosage at the orbit of STS-31 was about 200 milliRads/day. The total dosage was

118 hours x (1 day / 24 hours ) x 200 milliRads/day = 983 milliRads.

This equals about 983 milliRads/365 milliRads = 2.7 years of ground-level dosage

# The Deadly Van Allen Belts?

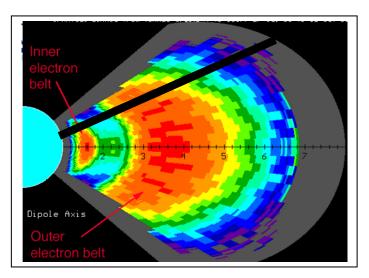


The numbers along the horizontal axis give the distance from Earth in multiples of the Earth radius (1 Re=6378 km). The Inner van Allen Belt is located at about 1.6 Re. The Outer van Allen Belt is located at about 4.0 Re. At a distance of 2.2 Re, there is a 'gap' region in between these belts. Satellites such as the Global Positioning System (GPS) orbit in this gap region where radiation effects are minimum.

The International Space Station and Space Shuttle, on this scale, orbit very near the edge of the blue 'Earth disk' in the figure, so are well below the Van Allen Belts.

In 1958, Dr. James Van Allen discovered a collection of high-energy particle clouds within 40,000 km of Earth. Arranged like two nested donuts, the inner belt is mainly energetic protons, while the outer belts contain both protons and electrons. These belts have long been known as 'bad news' for satellites and astronauts, with potentially deadly consequences if you spend too much time within them. The figure below, produced by scientists from the NASA, CRRES satellite, shows the radiation dosages at various locations within the belts.

Blue = 0.0001 Rads/sec Green= 0.001 Rads/sec Yellow= 0.005 Rads/sec Orange= 0.01 Rads/sec and Red= 0.05 Rads/sec.



Apollo astronauts, and astronauts in the upcoming visits to the Moon, will have to travel through some of these belt regions because the orbit of the Moon lies along the fastest line-of-travel from Earth. On the scale of the above figure, the distance to the Moon is 60 Re.

- 1. The speed of the spacecraft will be about 25,000 km/hour. If the spacecraft travels along the indicated path (black bar), how long, in minutes, will it spend in the Blue, Green, Yellow, Orange and Red regions?
- 2. Given the indicated radiation dosages in Rads/sec for each zone, what will be the dosages that the astronauts receive in each zone?
- 3. What will be the total radiation dosage in Rads for the transit through the belts?
- 4. Some people believe that the Apollo moon landings were a hoax because astronauts would have been instantly killed in the radiation belts. According to the US Occupation Safety and Health Agency (OSHA) a lethal radiation dosage is 300 Rads in one hour. What is your answer to the 'moon landing hoax' believers?

Note: According to radiation dosimeters carried by Apollo astronauts, their total dosage for the entire trip to the moon and return was not more than 2 Rads over 6 days.

Apollo astronauts, and astronauts in the upcoming visits to the Moon, will have to travel through some of these belt regions because the orbit of the Moon lies along the fastest line-of-travel from Earth. On the scale of the above figure, the distance to the Moon is 60 Re.

1. The speed of the spacecraft will be about 25,000 km/hour. If the spacecraft travels along the indicated path, how long, in minutes, will it spend in the Blue, Green, Yellow, Orange and Red regions? Note: transit estimates may vary depending on how accurately students measure figure.

Blue: 1.8 Re x (6378 km/Re) x (1 hour/25,000 km) x (60 minutes/1 hour) = 27.6 minutesYellow:  $(1.4 \times 6378) / 25,000 \times 60 =$  6.1 minutes Orange:  $(1.0 \times 6378) / 25,000 \times 60 =$  15.3 minutes Green:  $(0.25 \times 6378) / 25,000 \times 60 =$  3.8 minutes Red: 0 minutes

Total transit time...... 52.8 minutes

2. Given the indicated radiation dosages in Rads/sec for each zone, what will be the dosages that the astronauts receive in each zone?

Blue: = 27.6 minutes x ( 60 sec/ 1 minute) x (0.0001 Rads/sec) =	0.17 Rads
Yellow = 6.1 minutes x 60 sec/minute x 0.005 rads/sec =	1.83 Rads
Orange = 15.3 minutes x (60 sec/minute) x 0.01 rads/sec =	9.18 Rads
Green = 3.8 minutes x (60 sec/minute) x 0.001 rads/sec =	0.23 Rads

- 3. What will be the total radiation dosage in Rads for the transit through the belts? 0.17 + 1.83 + 9.18 + 0.23 = 11.4 Rads
- 4. Some people believe that the Apollo moon landings were a hoax because astronauts would have been instantly killed in the radiation belts. According to the US Occupation Safety and Health Agency (OSHA) a lethal radiation dosage is 300 Rads in one hour. What is your answer to the 'moon landing hoax' believers?

Note: According to radiation dosimeters carried by Apollo astronauts, their total dosage for the entire trip to the moon and return was not more than 2 Rads over 6 days.

The total dosage for the trip is only 11.4 Rads in 52.8 minutes. Because 52.8 minutes is equal to 0.88 hours, his is equal to a dosage of 11.4 Rads / 0.88 hours = 13 Rads in one hour, which is well below the 300 Rads in one hour that is considered to be lethal.

Also, this radiation exposure would be for an astronaut outside the spacecraft during the transit through the belts. The radiation shielding inside the spacecraft cuts down the 13 Rads/hour exposure so that it is completely harmless.

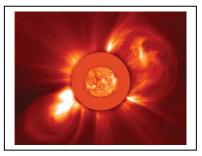
One of the most basic activities that scientists perform with their data is to look for correlations between different kinds of events or measurements in order to see if a pattern exists that could suggest that some new 'law' of nature might be operating. Many different observations of the Sun and Earth provide information on some basic phenomena that are frequently observed. The question is whether these phenomena are related to each other in some way. Can we use the sighting of one phenomenon as a prediction of whether another kind of phenomenon will happen?

During most of the previous sunspot cycle (January-1996 to June-2006), astronomers detected 11,031 coronal mass ejections, (CME: Top image) of these 1186 were 'halo' events. Half of these were directed towards Earth.

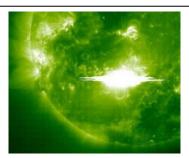
During the same period of time, 95 solar proton events (streaks in te bottom image were caused by a single event) were recorded by the GOES satellite network orbiting Earth. Of these SPEs, 61 coincided with Halo CME events.

Solar flares (middle image) were also recorded by the GOES satellites. During this time period, 21,886 flares were detected, of which 122 were X-class flares. Of the X-class flares, 96 coincided with Halo CMEs, and 22 X-class flares also coincided with 22 combined SPE+Halo CME events. There were 6 X-flares associated with SPEs but not associated with Halo CMEs. A total of 28 SPEs were not associated with either Halo CMEs or with X-class solar flares.

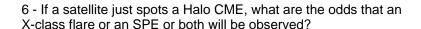
From this statistical information, construct a Venn Diagram to interrelate the numbers in the above findings based on resent NASA satellite observations, then answer the questions below.

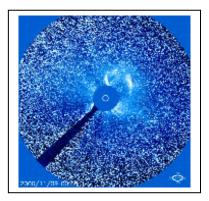


- 1 What are the odds that a CME is directed towards Earth?
- 2 What fraction of the time does the sun produce X-class flares?
- 3 How many X-class flares are not involved with CMEs or SPEs?



- 4 If a satellite spotted both a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur?
- 5 What percentage of the time are SPEs involved with Halo CMEs. X-class flares or both?

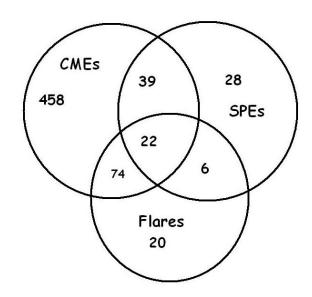




- 7 Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?
- 8 If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

Space Math

http://spacemath.gsfc.nasa.gov



Venn Diagram Construction.

- 1. There are 593 Halo CMEs directed to Earth so 593 = 74 with flares + 39 with SPEs + 22 both SPEs and Flares + 458 with no SPEs or Flares..
- 2. There are 95 SPEs. 95 = 39 with CMEs + 6 with flares + 22 with both flares and CMEs + 28 with no flares or CMEs
- 3. There are 122 X-class flares. 122 = 74 With CMEs only + 6 with SPEs only + 22 both CMEs and SPEs + 20 with no CMEs or SPEs.

- 1 What are the odds that a CME is directed towards Earth? 593/11031 = 0.054 odds = 1 in 19
- 2 What fraction of the time does the sun produce X-class flares? 122/21886 = **0.006**
- 3 How many X-class flares are not involved with CMEs or SPEs? 122 74 22 6 = 20.
- 4 If a satellite spotted BOTH a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur? 22/(74+22) = 0.23
- 5 What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?  $100\% \times (39+22+6/95) = 70.1\%$
- 6 If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?

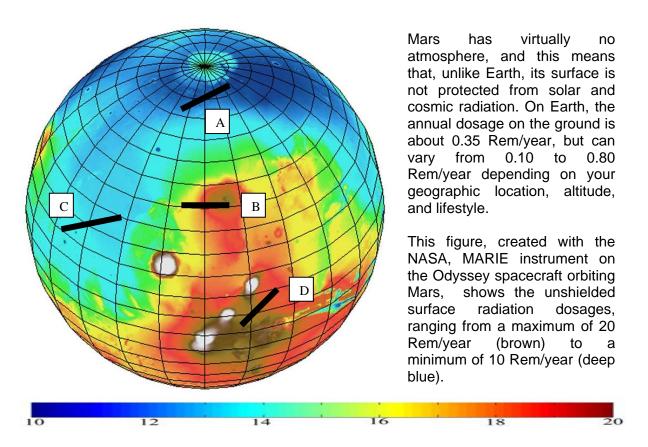
39+22+74 / 593 = 0.227 so the odds are 1/0.227 or about 1 in 4.

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed? (6+22)/95 = 0.295 or 1 out of 3 times for X-flares (39+22)/95 = 0.642 or 2 out of 3 for Halo CMEs

It is more likely to detect an SPE if a Halo CME occurs by 2 to 1.

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

39+6/95 = 0.50 so the odds are 1/0.50 or **2 to 1**.



Astronauts landing on Mars will want to minimize their total radiation exposure during the 540 days they will stay on the surface. The Apollo astronauts used spacesuits that provided 0.15 gm/cm<sup>2</sup> of shielding. The Lunar Excursion Module provided 0.2 gm/cm<sup>2</sup> of shielding, and the orbiting Command Module provided 2.4 gm/cm<sup>2</sup>. The reduction in radiation exposure for each of these was about 1/4, 1/10 and 1/50 respectively. Assume that the Mars astronauts used improved spacesuit technology providing a reduction of 1/8, and that the Mars Excursion Vehicle provided a 1/20 radiation reduction.

The line segments on the Mars radiation map represent some imaginary, 1,000 km exploration tracks that ambitious astronauts might attempt with fast-moving rovers, and not a lot of food! Imagine a schedule where they would travel 100 kilometers each day. Suppose they spend 20 hours a day within a shielded rover, and they study their surroundings in spacesuits for 4 hours each day.

- 1) Convert 10 Rem/year into milliRem/day.
- 2) What is the astronauts radiation dosage per day in a region (brown) where the ambient background produces 20 Rem/year?
- 3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the approximate total dosage to an astronaut in milliRems (mRems), given the exposure times and shielding information provided.
- 4) Which track has the highest total dosage in milliRems? The least total dosage? What is the annual dosage that is equivalent to these 20-day trips? How do these compare with the 350 milliRems they would receive if they remained on Earth?

1) Convert 10 Rem/year into milliRem/hour.

Answer:  $(10 \text{ Rem/yr}) \times (1 \text{ year/ } 365 \text{ days}) \times (1 \text{ day/24 hr}) = 1.1 \text{ milliRem/hour}$ 

2) What is the astronauts radiation dosage per day in a region (brown) where the background is 20 Rem/year? Answer: From Problem 1, 20 Rem/year = 2.2 milliRem/hour.

20 hours x (1/20) x 1.1 milliRem/hr + 4 hours x (1/8) x 1.1 milliRem/hr = 1.1 + 0.55 = 1.65 milliRem/day

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the total dosage in milliRems to an astronaut, given the exposure times and shielding information provided. The scaling relationship is that for each 20 Rems/year, the daily astronaut dosage is 0.66 milliRem/day (e.g. 0.66/20). The factor of 2 in the answers accounts for the round-trip.

Track A dosage:

 $2x(12 \text{ Rems/yr } x \ 10 \text{ days } x \ (1.65 / 20)) = 2x(9.9) = 19.8 \text{ mRem.}$ 

Track B dosage:

2x( 16 Rems/yr x 3.3days + 18 Rems/yr x 3.3days + 20 Rems/yr x 3.3days)(1.65/20) = 2x(14.8) = 29.6 mRem

Track C dosage:

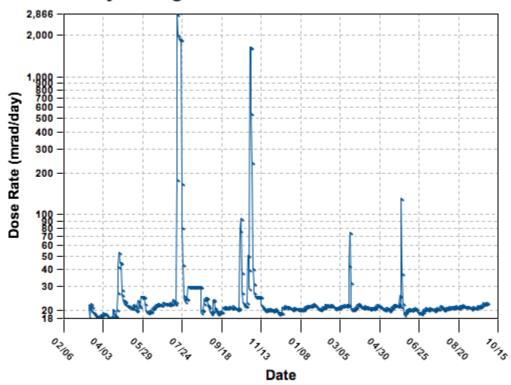
2x(12 Rems/yr x 5 days x (1.65/20) + 14 Rems/yr x 5 days x (1.65/20)) = 2x(5.0 + 5.8) = 21.6 mRem

Track D dosage:

2x(18 Rems/yr x 5 days x (1.65/20) + 20 Rems/yr x 5 days x (1.65/20)) = 2x(7.5 + 8.25) = 31.5 mRem

4) For this 20-day excursion, Track D has the highest dosage and Track A has the lowest. The equivalent annual dosage for the lowest-dosage track is 19.8 milliRem x 365 days/10 days = 722 milliRem, which is about twice the annual dosage they would receive if they remained on Earth. For the highest-dosage trip, the annualized dosage is 1,149 milliRems which is about 3 times the dosage on Earth.





The NASA, Mars Radiation Environment Experiment (MARIE) measured the daily radiation dosages from a satellite orbiting Mars between March 13, 2002 and September 30, 2003 as shown in the figure above. The dose rate is given in units of milliRads per day. (1 Rad = 2 Rems for cosmic radiation.) The six tall 'spikes' are Solar Proton Events (SPEs) which are related to solar flares, while the rest of the plotted data (the wiggly line!) is the dosage caused by galactic cosmic rays (GCRs).

- 1. By finding the approximate area under the plotted data, calculate the total radiation dosage in Rems for the GCRs during the observation period between 4/03/2002 and 8/20/2003.
- 2. Assuming that each SPE event lasted 3 days, and that its plotted profile is a simple rectangle, calculate the total radiation dosage in Rems for the SPEs during the observation period.
- 3. What would be the total radiation dosage for an unshielded astronaut orbiting Mars under these conditions?
- 4. Are SPEs more important than GCRs as a source of radiation? Explain why or why not in terms of estimation uncertainties that were used.

Teachers Note: Because students will be asked to determine the areas under a complicated curve using rectangles, please allow student answers to vary from the below estimates, by reasonable amounts! This may be a great time to emphasize that, sometimes, two scientists can get different answers to the same problem depending on how they do their calculation. Averaging together the student responses to each answer may be a good idea to improve accuracy!

1. By finding the approximate area under the plotted data, calculate the total radiation dosage in Rems for the GCRs during the observation period between 4/03/2002 and 8/20/2003.

From the graph, the average dosage rate is about 20 mRads/day. The time span is about 365 + 4x30 + 17 = 502 days. The area of a rectangle with a height of 20 milliRads/day and a width of 502 days is (20 milliRads/day) x (502 days) = 10040 milliRads. This can be converted to Rems by multiplying by (1 Rad/1000 milliRads) and by (2 Rem/1 Rad) to get **20 Rems.** 

2. Assuming that each SPE event lasted 3 days, and that its plotted profile is a simple rectangle, calculate the total radiation dosage in Rems for the SPEs during the observation period.

Peak 1 = 53 milliRads/day x 3 days = 159 millirads

Peak 2 = 2866 millirads/day x 3 days = 8598 milliRads

Peak 3 = 90 milliRads/day x 3 days = 270 milliRads

Peak 4 = 1700 milliRads/day x 3 days = 5100 milliRads

Peak 5 = 70 milliRads/day x 3 days = 210 milliRads

Peak 6 = 140 milliRads/day x 3 days = 420 milliRads

The total dosage is 14,757 milliRads.

Convert this to Rems by multiplying by (1 Rad/1000 milliRads) x (2 Rem/1 Rad) To get **30 Rems after rounding.** 

3. What would be the total radiation dosage for an unshielded astronaut orbiting Mars under these

Answer: 20 Rems + 30 Rems = 50 Rems for a 502-day visit.

4. Are SPEs more important than GCRs as a source of radiation? Explain why or why not.

**Answer:** Solar Proton Events may be slightly more important than Galactic Cosmic Radiation for astronauts orbiting Mars.

The biggest uncertainty is in the SPE dose estimate. We had to approximate the duration of each SPE by a rectangular box with a duration of exactly three days, although the plot clearly showed that the durations varied from SPE to SPE. If the average dose rate for each SPE were used, rather than the peak, and a shorter duration of 1-day were also employed, the estimate for the SPE total dosage would be significantly lower, perhaps by as much as a factor of 5, from the above estimates, which would make the GCR contribution, by far, the largest.

conditions?

# Single Event Upsets in Aircraft Avionics

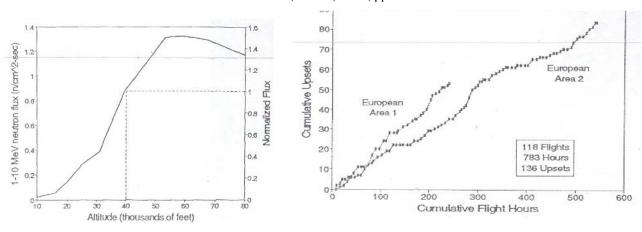


In 2001, the NASA Altair, Unmanned Air Vehicle (UAV) flew its first flight at an altitude of 100,000 feet. Designed by NASA-Dryden engineers and scientists, it is designed to fly for up to 48 hours to complete a variety of science research studies of Earth. For more details about this Dryden program, visit

#### http://www.nasa.gov/centers/dryden/news/FactSheets

Because of the complexity of the computer systems (called avionics) onboard, and the very high altitudes being flown, special attention had to be paid to cosmic ray showers. These particles, mostly neutrons, pass through the walls of the aircraft and can affect computer circuitry. For unmanned aircraft, the slightest computer glitch can spell the end of an \$8 million aircraft. This exercise explores some of the issues behind computer glitches at aircraft altitudes.

Between the high-radiation environment of space, and the comparative safety of the ground, lies the atmosphere. Most human activity in the military and commercial flight industry takes place between ground-level and 100,000 feet. As cosmic rays collide with atmospheric atoms, they liberate showers of particles deep into the lower atmosphere. The most penetrating of these are the charge-free neutrons. The two figures below show the neutron flux versus altitude, and data taken from aircraft flying at 29,000 feet. The data were taken from a research paper by Taber and Normand (1993), and published in the *IEEE Transactions on Nuclear Science*, vol. 40, No. 2, pp 120.



The left-hand curve gives the number of neutrons that pass through each square centimeter of surface every second (the neutron flux). The right-hand plot gives the cumulative number of memory upsets at an altitude of 30,000 feet after a given number of hours in the air.

- 1. What is the neutron flux at 30,000 feet? At 60,000 feet?
- 2. How many memory upsets were registered after 400 hours of flight?
- 3. If the aircraft carried 1560 memory modules (called SRAMS), each with 64,000 bytes of memory, how many bytes of memory were carried? How many binary 'bits' of memory were carried? (1 byte = 8 bits)
- 4. If each upset involved one bit having the wrong data value due to a neutron impact, how many bit upsets were registered per day?
- 5. If the area of each memory unit is  $7.5 \times 10^{-9}$  cm<sup>2</sup>, what is the total area of all the memory modules?
- 6. How many neutrons passed through this area in one second?
- 7. During the 400 hours of flight, how many neutrons passed through the memory modules?
- 8. What is the probability that one neutron will cause an upset?
- 9. How long do you have to wait for an upset to occur at 30,000 feet? At 60,000 feet? At 100,000 feet?

The left-hand curve gives the number of neutrons that pass through each square centimeter of surface every second (the neutron flux). The right-hand plot gives the cumulative number of memory upsets at an altitude of 30,000 feet after a given number of hours in the air.

1. What is the neutron flux at 30,000 feet?

From the graph: 0.35 neutrons/cm2/sec

How many memory upsets were registered after 400 hours of flight?From the graph: 60 upsets

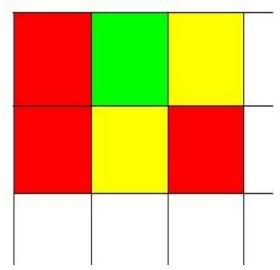
- 3. If the aircraft carried 1560 memory modules (called SRAMS), each with 64,000 bytes of memory, how many bytes of memory were carried? How many binary 'bits' of memory were carried? (1 byte=8 bits) 1560\*64000 = 99.8 megabytes x 8 bits/byte = 798,000,000 bits
- 4. If each upset involved one bit having the wrong data value due to a neutron impact, how many bit upsets were registered per day? 60/(400/24) = 3.6 bits/day for a population of 798,000,000 bits
- 5. If the area of each memory unit is  $7.5 \times 10^{-9} \text{ cm}^2$ , what is the total area of the memory modules?  $7.9 \times 10^{-9} \times 798,000,000 = 6.3 \text{ cm}^2$ .
- 6. How many neutrons passed through this total memory area in one second? From the answers to Problem 1 and 5:  $0.35 \text{ neutrons/cm}^2/\text{sec x } 6.3 \text{ cm}^2 = 2.2 \text{ neutrons/second.}$
- 7. During the 400 hours of flight, how many neutrons passed through the memory modules?
  2.2 neutrons/sec x 400 hours x 3600 seconds/hr = 3.2 million neutrons.
- 8. What is the probability that one neutron will cause an upset?

  From Problem 2 and 7: 60 upsets /3.2 million neutrons= 1 chance in 53,300.
- 9. How long do you have to wait for an upset to occur at 30,000 feet? The time it takes 53,300 neutrons to pass through the memory at 2.2 neutrons per second. 53,300/2.2 = 6.7 hours. Or you can get it by 400/60 = 6.7 hours.

At 60,000 feet, the neutron flux is 1.3/0.35 = 3.7 times higher than at 30,000 feet, so you would have to wait 6.7/3.7 = 1.8 hours.

At 100,000 feet, which is the cruising altitude of the Altair UAV, the graph suggests a neutron flux of about 1.0 neutrons/cm $^2$ /sec, so the flux is 1.0/0.35 = 2.9 times stronger at 100,000 feet than at 30,00 feet, and the time between upsets would be about 6.7/2.9 = 2.3 hours.

If the UAV were equipped with this much memory (about 100 megabytes) and was airborne for 48 hours, it would experience 48/2.3 = 21 memory upsets! This is why the computer systems on the UAV have to be radiation-hardened and the software designed to fix radiation errors when they occur.



The first few pixels in a large image

Data is sent as a string of '1's and '0's which are then converted into useful numbers by computer programs. A common application is in digital imaging. Each pixel is represented as a 'data word' and the image is recovered by relating the value of the data word to an intensity or a particular color. In the sample image to the left, red is represented by the data word '10110011', green is represented by '11100101' and yellow by the word '00111000', so the first three pixels would be word' transmitted as the 'three '101100111110010100111000'. But what if one of those 1-s or 0-s was accidentally reversed? You would get a garbled string and an error in the color used in a particular pixel.

Since the beginning of the Computer Era, engineers have anticipated this problem by adding a 'parity bit' to each data word. The bit is '1' if there are an even number of 1's in the word, and' 0' if there is an odd number. In the data word for red '10110011' the last '1' to the right is the parity bit.

When data is produced in space, it is protected by parity bits, which alert the scientists that a particular data word may have been corrupted by a cosmic ray accidentally altering one of the data bits in the word. For example, Data Word A '11100011' is valid but Data Word B '11110011' is not. There are five '1's but instead of the parity bit being '0' ('11100010'), it is '1' which means Data Word B had one extra '1' added somewhere. One way to recover the good data is to simply re-transmit data words several times and fill-in the bad data words with the good words from one of the other transmissions. For example:

Corrupted data string:	1011110 <b>0</b>	101101 <b>0</b>	1010101 <b>1</b>	0011001 <b>1</b>	1011101 <b>0</b>
Good data string:	1011110 <b>0</b>	100101 <b>0</b>	1010101 <b>1</b>	1011001 <b>1</b>	1011101 <b>0</b>

The second and fourth words have been corrupted, but because the string was re-transmitted twice, we were able to 'flag' the bad word and replace it with a good word with the correct parity bit. Cosmic rays often cause bad data in hundreds of data words in each picture, but because pictures are re-transmitted two or three times, the bad data can be eliminated and a corrected image created.

**Problem:** Below are two data strings that have been corrupted by cosmic ray glitches. Look through the data (a process called parsing) and use the right-most parity bit to identify all the bad data. Create a valid data string that has been 'de-glitched'.

10111010 01010000	11110101 01111010	101111100 10001100	11001011 00110111	00101101
01111000	11001101	10110111	11011010	11100001
10001010	10001111	01110011	10010011	11001011
10111010	01110101	10111100	11011011	10101101
01011010	01111010	10001000	10110111	00100110
11011000	11001101	10110101	11011010	11110001
10001010	10011111	01110011	10010001	11001011
	01010000 01111000 10001010 10111010 0101101	01010000 01111010 01111000 11001101 10001010 10001111 10111010 01110101 01011010 01111010 11011000 11001101	01010000 01111010 10001100 01111000 11001101 10110111 10001010 10001111 01110011 10111010 01110101 10111100 01011010 01111010 10001000	01010000       01111010       10001100       00110111         01111000       11001101       10110111       11011010         10001010       10001111       01110011       10010011         10111010       01110101       10111100       11011011         01011010       01111010       1001000       10110111         11011000       11001101       10110101       11011010

#### Answer Key:

**Problem:** Below are two data strings that have been corrupted by cosmic ray glitches. Look through the data (a process called parsing) and use the right-most parity bit to identify all the bad data. Create a valid data string that has been 'de-glitched'.

The highlighted data words are the corrupted ones.

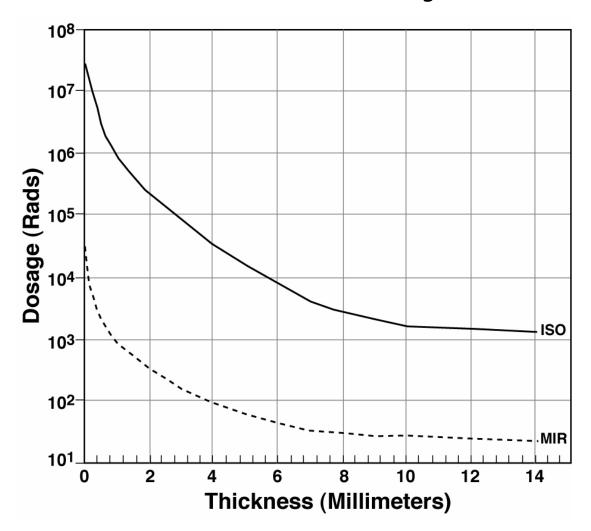
String 1:	10111010	11110101	10111100	11001011	00101101
_	01010000	01111010	10001100	00110111	00100110
	01111000	11001101	10110111	11011010	11100001
	10001010	10001111	01110011	10010011	11001011
String 2:	10111010	01110101	10111100	11011011	10101101
	01011010	01111010	10001000	10110111	10100110
	11011000	11001101	10110101	11011010	11110001
	10001010	10011111	01110011	10010001	01001011

In the first string, 11110101 has a parity bit of '1' but it has an odd number of '1' so its parity should have been '0' if it were a valid word. Looking at the second string, we see that the word that appears at this location in the grid is '01110101' which has the correct parity bit. We can see that a glitch has changed the first '1' in String 2 to a '0' in the incorrect String 1.

By replacing the highlighted, corrupted data words with the uncorrupted values in the other string, we get the following de-glitched data words:

Corrected:	10111010	01110101	10111100	11001011	10101101
	01011010	01111010	10001100	00110111	00100110
	11011000	11001101	10110101	11011010	11110001
	10001010	10001111	01110011	10010001	11001011

The odd word is the first word in the third row. The first transmission says that it is '01111000' and the second transmission says it is '11011000' Both wrong words have a parity of '1' which means there is an even number of '1' in the first seven places in the data word. But the received parity bit says '0' which means there was supposed to be an odd number of '1's in the correct word. Examining these two words, we see that the first three digits are '011' and '110' so it looks like the first and third digits have been altered. Unfortunately, we can't tell what the correct string should have been. Because the rest of the word '11000' has an even parity, all we can tell about the first three digits is that they had an odd number of '1's so that the total parity of the complete word is '0'. This means the correct digits could have been '100', '010', '111', or '111', but we can't tell which of the three is the right one. That means that this data word remains damaged and can't be de-glitched even after the second transmission of the data strings.



Satellites are designed to withstand many forms of radiation in the harsh environment of space. The above graph shows how the total life time radiation dosage inside a spacecraft changes as the amount of aluminum shielding increases. The data comes from the former MIR space station and the research satellite ISO. The sensitive instruments and electronic systems operate inside the satellite shell and are protected from harmful dosages of radiation by the shielding provided by the spacecraft walls.

**Problem 1**: You want to design a new satellite to replace the ISO satellite and to last 8 years in orbit, but it can only continue to work normally if it accumulates no more that 75,000 Rads of radiation during that time. Using the curve for ISO, how thick do the satellite walls have to be to insure this?

**Problem 2:** The International Space Station has the same orbit as the MIR. An astronaut will spend about 100 hours in space to assemble the station. If the equivalent shielding of her spacesuit is 1.0 mm of aluminum, how large of a dosage will she receive during this time? How does it compare to the 0.4 Rads she would receive if she stayed on the ground?

**Problem 3:** A cubical satellite has sides 1 meter across, and the density of the aluminum is 2.7 g/cc. How much mass, in kilograms, will the satellite have with 4 mm-thick walls? 12 mm-thick walls? If the launch cost is \$15,000 per kilogram, how much extra will it cost to launch the heavier, and bettershielded, satellite?

#### Answer Key:

**Problem 1**: You want to design a new satellite to replace the ISO satellite and to last 8 years in orbit, but it can only continue to work normally if it accumulates no more that 75,000 Rads of radiation during that time. Using the curve for ISO, how thick do the satellite walls have to be to insure this?

**Answer:** The annual dosage would be 75,000 rads/8 years = 9,375 rads/year. From the ISO curve, this level of radiation would occur with about 5.5 millimeters of aluminum shielding.

**Problem 2:** The International Space Station has the same orbit as the MIR. An astronaut will spend about 100 hours in space to assemble the station. If the equivalent shielding of her spacesuit is 0.5 mm of aluminum, how large of a dosage will she receive during this time? How does it compare to the 0.4 Rads she would receive if she stayed on the ground?

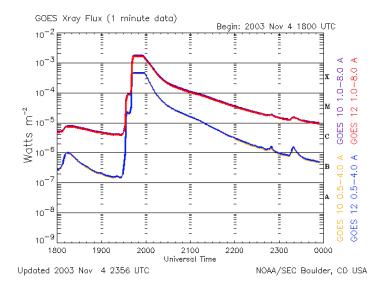
**Answer:** The graph shows that for 0.5 millimeters equivalent spacesuit thickness and a MIR orbit, the annual dosage is 800 Rads. But she will only spend 100 hours in space. There are 8760 hours in a year, so her actual dosage would be about 800 Rads/yr x (100 hrs/8760 hrs/yr) = 9.1 Rads. This is about 9.1/0.4 = 23 times the dosage she would get on the ground in one year..or equal to 23 years worth of dosage on the ground.

**Problem 3:** A cubical satellite has sides 1 meter across, and the density of the aluminum is 2.7 grame per cubic centimeter. How much mass, in kilograms, will the satellite have with 4 mm-thick walls? 12 mm-thick walls? If the launch cost is \$15,000 per kilogram, how much extra will it cost to launch the heavier, and better-shielded, satellite?

Answer: A) A cube consists of six sides. Each side has a volume of 1 meter x 1 meter x 4 millimeters, which in centimeters is  $= 100 \times 100 \times 0.4 = 4000$  cubic centimeters. The density of aluminum is 2.7 grams/cubic centimeter, so the mass of one side of the cube will be  $2.7 \times 4000 = 10,800$  grams or 10.8 kilograms. The entire satellite will have a mass of 6 x 10.8 kilograms or 64.8 kilograms.

- B) With 12-millimeter walls, the mass will be 100x100x1.2 x 2.7/1000 = 32.4 kilograms.
- C) The extra launch cost would be (32.4 10.8)x \$15,000/kg = \$324,000

### Solar Flare Reconstruction



During the November 4, 2003 solar flare, the GOES satellite measured the intensity of the flare as its light increased to a maximum and then decreased. The problem is that the solar flare was so bright that it could not record the most intense phase of the brightness evolution - what astronomers call its light curve. The figure above shows the light curve for two different x-ray energies, and you can see how its most intense phase near 19:50 UT has been clipped. This is a common problem with satellite detectors and is called 'saturation'. To work around this problem to recover at least some information about the flare's peak intensity, scientists resorted to mathematically fitting the pieces of the light curve that they were able to measure, and interpolated the data using their mathematical model, to estimate the peak intensity of the flare.

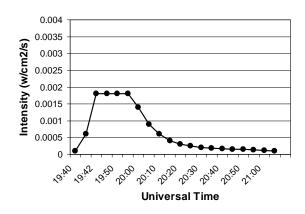
#### X-ray Flare Data.

Universal	Intensity
Time (UT)	(Watts/m <sup>2</sup> /sec)
19:40	0.00010
19:41	0.00060
19:42	0.00180
19:45	saturated
19:50	saturated
19:55	0.00180
20:00	0.00140
20:05	0.00090
20:10	0.00060
20:15	0.00040
20:20	0.00030
20:25	0.00025
20:30	0.00020
20:35	0.00019
20:40	0.00017
20:45	0.00016
20:50	0.00015
20:55	0.00014
21:00	0.00012
21:05	0.00010

<u>Problem 1 (Pre-Algebra):</u> Re-plot the data in the table and from the trend on either side of the saturated region, estimate the peak intensity.

<u>Problem 2 (Algebra):</u> Re-plot the data, and from the information on either side of the saturation region, create two exponential functions that fit the data. Use the elapsed time since 19:40 as the independent variable. Find the intersections of these two functions to estimate the peak intensity and time.

<u>Problem 3 (Calculus):</u> Integrate the piecewise function in Problem 2 to determine the area under the light curve to 21:05. Note: 1 Watt equals 1 Joule of energy per second. Given that the sun is 147 million kilometers from the GOES satellite, calculate the surface area, in square meters, of a sphere of this radius. Calculate the total energy, in Joules, radiated by the flare that passed through the surface area of the sphere.



Re-plotted data to left, allowing extra space for interpolation.

#### Problem 1 (Pre-Algebra):

Answer: The curves, drawn free-hand, intersect between 0.0035 to 0.004 Watts/m<sup>2</sup>/sec

**Problem 2 (Algebra):** Create two exponential functions that fit the data. Use the elapsed time since 19:40 as the independent variable.

Rising: From (0.0, 0.0001), (1.0, 0.0006) and (2.0, 0.00018) a best-fit exponential curve is  $R(T) = 0.0001 e^{(+1.44T)}$ 

Falling: From (20.0, 0.0018), (25.0, 0.0014), (30.0, 0.0009) and (35.0, 0.0006) a best-fit exponential curve is  $R(T) = 0.0074 e^{(-0.07T)}$ 

Find the intersections of these two functions to estimate the peak intensity and time.

 $0.0001 e^{1.44T} = 0.0074 e^{-0.07T}$ 

Taking  $\log_{\mathbf{e}}$  of both sides :  $\log_{\mathbf{e}}(0.0001) + 1.44 \text{ T} = \log_{\mathbf{e}}(0.0074) - 0.07T$ solve for T to get: T = (+9.21 - 4.91)/1.51 = 2.84 minutes So the peak UT is 19:40 + 2.84 = 19:42.84 or 19:42:50

The peak intensity is then  $0.0001 e^{(1.44 \times 2.84)} = 0.006 \text{ Watts/m}^2$ 

**Problem 3 (Calculus):** Integrate the piecewise function in Problem 2 to determine the area under the light curve. Note 1 watt x 1 second = 1 Joule.

Rising-side: From 0 to 2.84 minutes: 0.0001 x (1/1.44) [ e (1.44 x 2.84) - 1 ] = 0.0041 Joules/m  $^2$ 

Falling side from 2.84 to 85 minutes:

Combining we get a total 'flux' of 0.091 Joules/m<sup>2</sup>

Given that the sun is 147 million kilometers from the GOES satellite, calculate the surface area of a sphere of this radius. Calculate the total energy radiated by the flare in ergs that passed through the surface area of the sphere.

Area = 
$$4 \pi (147 \times 10^6 \text{ km})^2$$
 =  $2.71 \times 10^{17} \text{ km}^2 \times 1.0 \times 10^6 \text{ meter}^2/\text{km}^2$  =  $2.71 \times 10^{23} \text{ m}^2$   
Total energy =  $0.091 \text{ Joules/m}^2 \times 2.71 \times 10^{23} \text{ m}^2$  =  $2.5 \times 10^{22} \text{ Joules}$ 

Space Math

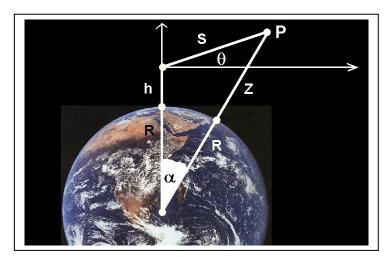
http://spacemath.gsfc.nasa.gov

The least expensive form of radiation shielding is a planetary atmosphere, but just how efficient is it? The walls of the International Space Station and the Space Shuttle provide substantial astronaut protection from space radiation, and have an equivalent thickness of 10 grams/cm<sup>2</sup> of aluminum, which has a density of 2.7 grams/cm<sup>3</sup>. Compare this shielding to the spacesuits worn by Apollo astronauts of only 0.1 grams/cm<sup>2</sup>. The atmosphere of Earth is a column of air with density of 0.0012 grams/cm<sup>3</sup>, that is 100 kilometers tall. How much shielding does this provide at different altitudes above the ground?

In this three-part problem, we will begin the first step in constructing a mathematical model of the shielding from a planetary atmosphere. A similar calculation was published by Drs. Lisa Simonsen and John Nealy in February, 1993 in the article "Mars Surface Radiation Exposure for Solar Maximum Conditions and 1989 Solar Proton Events", (NASA Technical Paper 3300)

The figure below right gives the necessary geometry and variable definitions.





The figure shows a radiation sampling point located 'h' above Earth's surface, and radiation from a source at point P, which is located at a distance 'S' from the sampling point. The distance from Earth's surface to point P is given by 'z'. Also, as seen from the sampling point, the vertical arrowed ray points to a point straight overhead, and the horizontal arrowed ray points to the horizon. The angle ' $\theta$ ' is the elevation angle of the radiation source from the sampling point. So, a scientist would place a radiation detector at the sampling point located above Earth's surface, point the instrument at the radiation source at point P, and make a measurement of the amount of radiation coming from that particular direction in the sky.

Problem 1: From the information given in the figure, calculate the distance, S, in terms of h, R, z, and  $\theta$ .

Problem 2: What is the form of  $S(R, h, z, \theta)$  when;

- A) If h is very much smaller than R? (h approaches zero)
- B)  $\theta = 90^{\circ}$  ?
- C) If z is very much smaller than R? (z approaches zero)

Problem 1: From the information given in the figure, calculate the distance, S, in terms of h, R, z, and  $\theta$ . First, take three deep breaths, and play with the figure a bit. After some fascinating trial-and-error attempts, the simplest thing to realize is that the Law of Cosines can be used. There is only one of the three forms of this Law that do not involve the undesired angle,  $\beta$ , namely:

$$(R + Z)^2 = S^2 + (R + h)^2 - 2 (R + h) S cos(\theta + 90^0)$$

Where we can use the angle addition theorem,  $\cos{(A+B)} = \cos(A)\cos(B) - \sin(A)\sin(B)$  to simplify it:  $(R+Z)^2 = S^2 + (R+h)^2 + 2(R+h)S\sin(\theta)$ 

At first, it doesn't look like this pile of junk is useful because S doesn't appear by itself on one side of the equals sign. But by re-arranging, you see that it is really an equation with an interesting form:

$$S^2 + [2(R+h)\sin(\theta)] + [(R+h)^2 - (R+Z)^2] = 0$$
 which is a quadratic equation in which the coefficients are  $A = 1$   $B = 2(R+h)\sin(\theta)$  and  $C = (R+h)^2 - (R+Z)^2$ 

We use the quadratic equation to solve for the positive root, because the negative root has no physical meaning. With a 'little' algebra we get:

**Answer --- >** 
$$S(R,h,z,\theta) = ((R+h)^2 \sin^2 \theta + 2R(z-h) + z^2 - h^2)^{1/2}$$
 -  $(R+h) \sin \theta$ 

Problem 2: What is the form of  $S(R, h, z, \theta)$  when;

A) h << R ? Answer: Let h = 0 
$$S(R,h,z,\theta) = \left( R^2 \sin^2 \theta + 2 R z + z^2 \right)^{1/2} - R \sin \theta$$

B) 
$$\theta = 90^{\circ}$$
 ? Answer:

$$S(R,h,z,\theta) = ((R+h)^2 + 2R(z-h) + z^2 - h^2)^{1/2} - (R+h)$$

We can simplify this as

$$S(R,h,z,\theta) = \left( (R+h)^2 + 2R(z-h) + z^2 - h^2 \right)^{1/2} - (R+h)$$

$$S(R,h,z,\theta) = \left( R^2 + 2Rh + h^2 + 2Rz - 2Rh + z^2 - h^2 \right)^{1/2} - (R+h)$$

$$S(R,h,z,\theta) = \left( R^2 + 2Rz + z^2 \right)^{1/2} - R - h$$

$$S(R,h,z,\theta) = (R+z) - R - h$$

**Answer --- >** 
$$S(R,h,z,\theta) = z - h$$

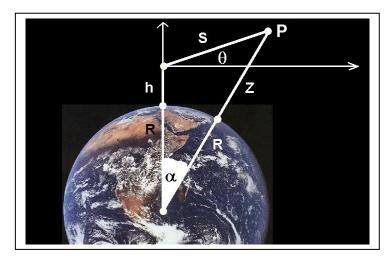
C)  $z \ll h$ ? Answer: Set z = 0 then

$$S(R,h,z,\theta) = ((R + h)^2 \sin^2 \theta + 2Rh - h^2)^{1/2} - (R + h) \sin \theta$$

The least expensive form of radiation shielding is a planetary atmosphere, but just how efficient is it? The walls of the International Space Station and the Space Shuttle provide substantial astronaut protection from space radiation, and have an equivalent thickness of 10 grams/cm<sup>2</sup> of aluminum, which has a density of 2.7 gm/cm<sup>3</sup>. Compare this shielding to the spacesuits worn by Apollo astronauts of only 0.1 gm/cm<sup>2</sup>. The atmosphere of Earth is a column of air with density of 0.0012 gm/cm<sup>3</sup>, that is 100 kilometers tall. How much shielding does this provide at different altitudes above the ground?

In the previous problem 'Atmospheric Shielding from Radiation I' we defined a function that gives the length of the path from the radiation source to the measurement point located h above Earth's surface. To find the amount of shielding provided by the atmosphere, we have to multiply this length, by the density of the atmosphere along the path S. In this problem, we will assume that the atmosphere has a constant density of 0.0012 grams/cm $^3$ , and see what the total shielding is along several specific directions defined by  $\theta$ .





The formula for S is given by:

$$S(R,h,z,\theta) = \left( (R+h)^2 \sin^2 \theta + 2 R (z-h) + z^2 - h^2 \right)^{1/2} - (R+h) \sin \theta$$

Assume R = 6,378 kilometers.

Problem 1: What is the form of the function that gives the shielding for a direction A) straight overhead ( $\theta = 90^{\circ}$ ) and B) at the horizon ( $\theta = 0^{\circ}$ ), for a station at sea-level (h=0 kilometers)?

Problem 2: More than 90% of the atmosphere is present below an altitude of about 2 kilometers. If this is approximated as being uniform in height, what is the total shielding towards the zenith (overhead) and the horizon, if z = 2 kilometers?

Problem 3: The atmosphere of Mars is about 100 times less dense, and mostly resides below 1 kilometer in altitude. Re-calculate the answers to Problem 2, and compare the radiation dosage difference at the surface of each planet.

The formula for S is given by:

$$S(R,h,z,\theta) = ((R+h)^2 \sin^2\theta + 2R(z-h) + z^2 - h^2)^{1/2} - (R+h) \sin\theta$$

Shielding  $D(R,h,z,\theta) = 0.0012 \times S(R,h,z,\theta)$  in units of gm/cm<sup>2</sup> for S given in cm.

Problem 1: What is the form of the function that gives the shielding for A) a direction straight overhead ( $\theta = 90^{\circ}$ ), and B) at the horizon ( $\theta = 0^{\circ}$ ), for a station at sea-level (h=0 kilometers)?

Answer: A)  $D = 0.0012 \times Z$  where Z is in centimeters. B)  $D(R,h,z,\theta) = 0.0012 \left( z^2 + 2 R z \right)^{1/2}$  where R and z are in centimeters.

Problem 2: More than 90% of the atmosphere is present below an altitude of about 2 kilometers. If this is approximated as being uniform in height, what is the total shielding towards the zenith (overhead) and the horizon, if z = 2 kilometers?

Answer:

- A)  $D = 0.0012 \text{ gm/cm}^3 \times 200,000 \text{ cm} = 240 \text{ gm/cm}^2$  for radiation entering from straight overhead.
- B) Because  $z \ll R$ ,  $z^2 \ll 2Rz$  so

D = 
$$0.0012 \times (2 \text{ R z})^{1/2}$$
  
=  $0.0012 \times (2 \times (2 \times 10^5) \times (6.278 \times 10^6))^{1/2}$   
=  $0.0012 \times 1.59 \times 10^6$   
=  $1916 \text{ gm/cm}^2$  for radiation entering from the horizon direction

Problem 3: The atmosphere of Mars is about 10 times less dense. Re-calculate the answers to Problem 2, and compare the radiation dosage difference at the surface of each planet.

Answer: For mars, R = 3,374 km, density = 0.00012 gm/cm<sup>3</sup> then from Problem 2:

- A)  $D = 0.00012 \text{ gm/cm}^3 \text{ x } 100,000 \text{ cm} = 12 \text{ gm/cm}^2$
- B) D =  $0.00012 \times (2 R z)^{1/2}$ =  $0.00012 \times (2 \times (1 \times 10^5) \times (3.374 \times 10^6))^{1/2}$ =  $0.00012 \text{ gm/cm}^3 \times 8.2 \times 10^5 \text{ cm}$ =  $98 \text{ gm/cm}^2$

The minimum radiation shielding comes from directions above your head that pass through the least amount of atmosphere. The amount of radiation shielding at the surface of Mars is  $(240 \text{ gm/cm}^2)/(12 \text{ gm/cm}^2) = 20 \text{ times less}$  than on Earth. That means that radiation dosages at the surface of Mars would be about 20 times higher than on Earth's surface. Instead of 27 mRems/year, which is typical of the cosmic ray background on Earth's surface, you would receive about  $27 \times 20 = 560 \text{ mRems/year}$  on Mars. Compare this with 370 mRems/year as the average human dosage on Earth from all sources.

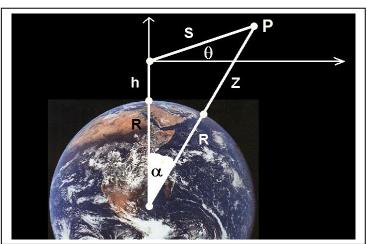
In the next problem 'Atmospheric Shielding from Radiation III' we will calculate this shielding more exactly.

# Atmospheric Shielding from Radiation III

The least expensive form of radiation shielding is a planetary atmosphere, but just how efficient is it? The walls of the International Space Station and the Space Shuttle provide substantial astronaut protection from space radiation, and have an equivalent thickness of 10 grams/cm<sup>2</sup> of aluminum, which has a density of 2.7 gm/cm<sup>3</sup>. Compare this shielding to the spacesuits worn by Apollo astronauts of only 0.1 gm/cm<sup>2</sup>. The atmosphere of Earth is a column of air with density of 0.0012 gm/cm<sup>3</sup>, that is 100 kilometers tall. How much shielding does this provide at different altitudes above the ground?

In the previous problem 'Atmospheric Shielding from Radiation II' we estimated the atmospheric shielding of Earth and Mars and compared the potential radiation dosages on the planetary surface. In this problem, we will create a more accurate estimate by using a realistic model for the atmospheres of these planets. Assume R (Earth) = 6,378 kilometers, R (Mars = 3,374 km)





The formula for S is given by

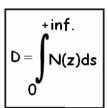
$$S(R,h,z,\theta) = ((R + h)^2 \sin^2 \theta + 2 R (z - h) + z^2 - h^2)^{1/2} - (R + h) \sin \theta$$

**Problem 1:** What is the form of the function S for h=0?

The density of a planetary atmosphere is defined by the exponential function N(z) = N(0) e (-Z/H) where H is the scale-height of the gas. For the composition of Earth's atmosphere, temperature, and surface gravity, H = 8.5 km. For Mars, H = 11.1 km. The sea-level density for Earth, N(0) = 0.0012 gm/cm<sup>3</sup>, while for Mars, N(0) = 0.00020 g/cm<sup>3</sup>. The amount of surface shielding for radiation arriving from a direction,  $\theta$ , is given by evaluating the integral below:

#### Problem 2:

- A) Determine the form for S for the case of  $\theta$  = 90 which gives the minimum planetary shielding at the surface for radiation entering from directly overhead.
- B) Evaluate the integral for Earth and for Mars.
- C) Assuming that the radiation environments of Mars and Earth are otherwise similar, about how many times more would your radiation dosage be on the surface of Mars compared to Earth?
- D) How does the atmospheric shielding of Earth compare to the shielding provided by the International Space Station or the Space Shuttle?



The formula for S is given by

$$S(R,h,z,\theta) = ((R + h)^2 \sin^2 \theta + 2 R (z - h) + z^2 - h^2)^{1/2} - (R + h) \sin \theta$$

**Problem 1:** What is the form of the function S for h=0?

$$S(R, z, \theta) = (R^2 \sin^2 \theta + 2Rz + z^2)^{1/2} - R \sin \theta$$

**Problem 2:** The density of a planetary atmosphere is defined by the exponential function  $N(z) = N(0) e^{(-Z/H)}$  where H is the scale-height of the gas. For the composition of Earth's atmosphere, temperature, and surface gravity, H = 8.5 km. For Mars, H = 11.1 km. The sea-level density for Earth,  $N(0) = 0.0012 \text{ gm/cm}^3$ , while for Mars,  $N(0) = 0.00020 \text{ g/cm}^3$ .

A) From the definition of S in Problem 1, determine the form for S for the case of  $\theta$  = 90 which gives the minimum planetary shielding at the surface for radiation entering from directly overhead.

Answer: 
$$\sin(90) = 1$$
 so  $S = (R^2 + 2Rz + z^2)^{1/2} - R$   $S = (R+z) - R$  so  $S = z$  !!!

B) Evaluate the integral for Earth and for Mars.

From A) s = z so by substituting s for z, the integral becomes ......

$$D = \int_{0}^{+\inf} N(0) e^{-(z/H)} dz$$

Using the variable substitution x = z/H, you can put the integrand in a standard form......

 $D = N(0) H \int_{0}^{+inf.} e^{-x} dx$ 

Evaluating the integral.....

$$D = N(0) H (e^{-0} - e^{-(infinity)})$$

The answer is that

$$D = N(0) H$$

For Earth:  $D = 1.2 \text{ kg/m}^3 \text{ x } 8.5 \text{ km} = 1,020 \text{ gm/cm}^2$ For Mars:  $D = 0.020 \text{ kg/m}^3 \text{ x } 11.1 \text{ km} = 22 \text{ gm/cm}^2$ 

C) Assuming that the radiation environments of Mars and Earth are otherwise similar, about how many times more would your radiation dosage be on the surface of Mars compared to Earth?

**Answer:** Note: This means that, because your maximum radiation dosage comes from radiation reaching you from the vertical direction (less shielding), on Mars, you will be receiving about 1,020 gm/cm $^2$  / 22 gm/cm $^2$  or 46 times as much radiation on the ground as you would get on Earth. On Earth, your annual cosmic ray dosage is about 27 mRem /year, so on Mars the dosage could be 46 x 0.027 Rem/year = 1.2 Rem/year.

D) How does the atmospheric shielding of Earth compare to the shielding provided by the International Space Station or the Space Shuttle?

**Answer:** The ISS shielding is about 10 gm/cm<sup>2</sup>, but the atmospheric shielding on the ground for Earth is 1,020 gm/cm<sup>2</sup> which is100 times greater!

## A note from the Author,

Radiation is one of those topics that have been a source of public concern ever since the first 'atom bomb' was detonated. Before the Atmospheric Test Ban Treaty was signed in the mid 1960's, hundreds of nuclear bombs were detonated above ground, spewing forth kilotons of radioactive dust and debris that took up temporary residence in the atmosphere. Cows ate grasses and produced milk with high levels of strontium-90, while other isotopes of iodine and potassium also made their way into our food, at least for a decade or two. Then came the development of nuclear power plants, the accidents at Three Mile Island and Chernobyl. Non-nuclear sources of radiation also became a growing public concern, including high-voltage power lines and cell phones. So, it is not surprising that the Public, through numerous media reports, accidents, and scientific studies, have learned to be wary of 'radiation' and to consider it not only bad in all forms, but something that our Government should strictly regulate.

The problem is that radiation exposure is a biologically complex subject that cannot be described in a few sound-bites or slogans. Like it or not, we exist in a sea of radiation that we can do nothing about. Cosmic rays, natural radioactivity in our soils and clays, and radon gas in our basements, make up 80% of our natural background dose, and cannot be eliminated. Even the bananas we eat give us a measurable dosage of radioactivity.

Because radiation exposure is so important to NASA, the health of its astronauts, and operation of sensitive equipment, this book is dedicated to de-mystifying 'radiation' through concrete mathematical exercises. I hope that you and your students gain a better appreciation of this subject, and learn to think about it with increased clarity!

Sten Odenwald

Astronomer

Space Math @ NASA

